Information and Preferences in Shareholder Voting[*]

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Abstract

We develop a model of shareholder voting with incomplete information about proposal quality. Shareholders differ in their ownership stake, private information precision, and unconditional preference towards proposals passing. Equilibrium voting makes the mapping between observed vote records and preferences ambiguous because of strategic voting (shareholders conditioning on information implied by others' votes) and belief correlation (shared public signals about proposal quality). We estimate the model using voting records of large US mutual funds and find that blockholders' observed support rates can differ considerably from preference-implied support rates, highlighting that investor preferences cannot be inferred directly from vote records.

Keywords: corporate governance, shareholder voting, institutional investors, passive investing, strategic voting

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1. Introduction

Institutional investors occupy blockholder positions in practically all listed firms in the US. Through ownership and systematic participation in shareholder voting, these investors may exert substantial influence on corporate governance. Research has shown that large institutional blockholders exhibit higher propensities to support management in shareholder voting than do smaller shareholders, and these differences may be interpreted as evidence of preference towards management. Implicit in this interpretation is that support rates are sufficient statistics for (institutional) investor preferences.

An important point is that investors vote without observing a proposal's true value; a rational investor forms beliefs about whether the proposal passing will increase or decrease firm value by observing public information (via proxy statements and advisor recommendations, for example), their own private information, and information embedded in the voting environment itself. As a result, the same investor may vote for or against different proposals that differ from their unconditional preference towards that proposal passing.

In this paper, we develop a model in which firms are held by blockholders and small dispersed shareholders who vote on proposals put forward by management. Some proposals increase firm value while others decrease it, but the exact effect is realized only after the vote. A shareholder may have a preference for passage, captured by an additional payoff if the proposal passes, regardless of its fundamental value. In equilibrium, each shareholder supports the proposal when the private signal exceeds an endogenously chosen, shareholder-specific threshold (i.e., a cutoff strategy, with lower thresholds implying higher support rates.) We study how blockholders' preferences for passage and the informational environment together determine these thresholds and, in turn, observed voting.

A stylized example clarifies the mechanism. Suppose we observe many management proposals across firms with a common blockholder and many dispersed shareholders, and we see that the blockholder supports more proposals than smaller shareholders. This implies that the blockholder supports some proposals even when it receives a low signal, and one might conclude that the blockholder receives extra payoff when the payoff passes to offset potential decreases in firm value, i.e., the blockholder is biased towards management.

This reasoning is incomplete because the blockholder votes strategically, by anticipating the states of the world in which she will swing the outcome of the vote. Therefore, the blockholder ought to vote as if the support rate is close to the passing threshold: as if she is pivotal. The event of being pivotal itself is a signal about the proposal's quality, which the blockholder incorporates into their decision. Because the remaining shareholders vote conservatively, supporting fewer proposals than the blockholder on average, they wait for a (relatively) high signal before voting for the proposal. When the blockholder is pivotal, she rationally acts as if she also received a high signal. This *strategic effect* inherent in voting games means that a shareholder's observed equilibrium voting behavior is not only a function of her own information and preferences, but also those of other shareholders. Importantly, this equilibrium strategic interdependence differs from the *signal effect*, which impacts the blockholder's through her own private information and shared public information.

Given these effects, what can we infer about the blockholder's preference towards supporting management proposals? If the remaining shareholders' signals are uncorrelated, the strategic effect is equivalent to the blockholder receiving a precise signal that the proposal increases firm value: an individual shareholder might vote for a bad proposal idiosyncratically, but the probability that most of them make the same error is small. Therefore, the strategic effect can offset the signal effect. Even though the blockholder supports some proposals that its private signal suggests are more likely to be bad (the signal effect), this is offset by the fact that when she is pivotal, the proposal is more likely to be good (the strategic effect). In this case, the blockholder may not be biased: it rationally supports more proposals because when its vote matters, the proposal is likely to increase firm value.

In practice, shareholders' information is correlated. For example, both small and large shareholders rely on proxy advisor recommendations to shape their voting decisions. We allow this to enter our model by letting all shareholders see a common signal about proposal quality. Importantly, strategic effects attenuate when signals are highly correlated. In the example above, a precise common proxy advisor signal means that the votes of other shareholders convey less information. The blockholder places more weight on its own signal, and since it waits only for a low signal before supporting management, it votes for some proposals likely to decrease value. This implies that the blockholder is biased, because the additional payoff it earns when the proposal passes offsets these bad proposals.

These observations imply that comparing blockholder support rates to those of smaller shareholders can lead to incorrect inferences about preferences. An unbiased blockholder can appear more supportive because of information aggregation in the state when she is pivotal, while a blockholder with a preference for passage can appear similar to others when signals are highly correlated. These ambiguities extend to the more empirically relevant setting with multiple blockholders. For credible inference on shareholder preferences, an estimator must correct for strategic and informational interactions.

We estimate our model using data on compensation proposals and recover latent preferences and information parameters. The estimation follows a two-step approach standard in the industrial organization literature. We first estimate investor-specific voting strategies and treat these as the equilibrium. We then invert best-response conditions to recover the set of preference and information parameters that rationalize the estimated strategies. Because this inversion conditions on the strategies of the rest of the voting base, the recovered preferences are adjusted for strategic effects and the correlation structure of shareholder information.

In the raw data, the "Big Three," Vanguard, BlackRock, and State Street, display higher unconditional support rates for management proposals than dispersed shareholders. Across all proposals, their average approval frequencies range from 90% to 95%, compared to 82% among dispersed shareholders. Conditional on ISS recommendations, however, these differences narrow sharply: when ISS recommends "For," nearly all investors support management, whereas when ISS recommends "Against," blockholders vote against management with non-trivial frequency. This empirical pattern, which we exploit in our estimation strategy suggests that raw voting differentials conflate both preference heterogeneity and information effects. Large investors may interpret the same public signals differently or receive more precise private information, rather than simply holding stronger pro-management views.

Our estimation procedure yields consistent estimates for preferences towards management proposals of the Big Three. Vanguard and BlackRock are statistically indistinguishable from dispersed shareholders in their underlying stance toward management, while State

Street appears slightly more favorable, which differs markedly from the observed support rates in the previous paragraph.

Importantly these estimates are corrected for strategic effects and correlated information across shareholders. We undertake a decomposition analysis that allows us to precisely apportion observed support rates into impacts from preferences, private information, public information and strategic behavior. The analysis shows that once strategic and informational effects are taken into consideration, (counterfactual) preference-implied support rates of blockholders would be close to or even less than that of dispersed shareholders.

Our empirical results highlight the model's central mechanism, higher observed support for proposals does not necessarily represent a stronger preference towards passing: strategic pivotality and information asymmetry together generate an equilibrium in which investors with more precise signals or greater voting weight may appear more supportive towards management even when their underlying preferences may not suggest the same.

The paper is organized as follows. The rest of this section outlines the paper's contribution within the literature. Section 2 outlines the voting model of shareholder voting with incomplete information and Section 3 presents numerical examples to develop intuition about the model. Section 4 gives the estimation strategy and Section 5 presents results of the estimation. Finally, Section 6 concludes.

Related Literature. This paper contributes to debates about how large diversified investors affect corporate governance, and to the study of voting under incomplete information. Legal and governance scholarship offers different views of large passive institutions. Bebchuk and Hirst (2019) argue that low-fee business models and potential conflicts can tilt preferences toward passage. Kahan and Rock (2020) and Fisch et al. (2019) emphasize that scale can strengthen stewardship incentives. The analysis here complements these perspectives by showing that observed support rates need not map into preferences once strategic pivotality and information aggregation are taken into account.

The paper relates to theoretical models of voting with common values and private signals, including Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1997), and Levit and Malenko (2011). We incorporate blockholder ownership and allow for a preference-for-passage component, and We study how correlated signals attenuate the informativeness of the pivotal event. The interpretation of correlation follows Malenko and Malenko (2019), who analyze the effect of proxy-advisor recommendations on information aggregation. Bar-Isaac and Shapiro (2020) examine participation versus abstention with blockholders; Levit et al. (2025) study trading before the vote with public information; Malenko et al. (2025) consider the design of proxy-advisor products. My focus is on how preferences and information jointly determine equilibrium cutoff strategies and observed support.

Empirical work has sought to measure investor philosophies from votes. Bubb and Catan (2022) use principal components to identify dimensions of governance preferences; Bolton et al. (2020) place mutual fund families along an ideological spectrum; Yi (2021) uses a Bayesian approach to characterize preferred governance structures. My approach is complementary. We recover preference-for-passage and information parameters by estimating equilibrium strategies and inverting best responses, which directly adjusts for strategic and informational interactions.

Finally, we relate to structural work on turnout in shareholder voting. Zachariadis et al. (2020) estimate a model of participation decisions in large electorates and study strategic effects in turnout. We treat turnout as exogenous and focus on how private information, correlated signals, and payoff shifters map into cutoff strategies and support rates. Both margins are important for corporate governance, and the identification strategies differ because the economic questions differ.

2. Model

This section outlines the model. The framework is deliberately parsimonious: we consider ownership of a single firm, held by multiple investors, and voting on a single management proposal of uncertain quality. This minimal structure allows us to isolate the informational and strategic forces that shape shareholders' voting behavior. Later, when we estimate the model, we extend it to a panel of firms and proposals.

2.1. Voting Environment

Proposal and State. Each firm j is owned by a set of investors who may each hold one or more shares. The firm's management puts forward a proposal of unknown quality, such as an advisory vote on executive compensation or the election of a director, and shareholders vote to determine whether it passes. Proposal quality is captured by a common-value state

$$x_j \sim N(0,1), \tag{1}$$

which is unknown to shareholders at the time of voting but has a prior distribution that is common knowledge. Management provides a recommendation for the proposal; voting in favor corresponds to supporting management's recommendation.

Preferences. Each shareholder i is characterized by a preference parameter $\delta_i \in \mathbb{R}$, which we interpret as a private value attached to the proposal passing. Let $\Delta \equiv \{\delta_i\}$ denote the (common knowledge) set of preferences for all shareholders. Shareholders with higher δ_i receive greater utility when the proposal passes. We interpret investors with positive δ_i as being more inclined to support management; they favor the passage of proposals regardless of their intrinsic quality. However, a positive δ_i simply captures an investor's preference for proposals to pass and need not reflect any literal bias toward management.

Payoffs. Let i denote a shareholder, whether a blockholder or a dispersed shareholder. Given the outcome of the vote $\mathrm{P}_j \in \{0,1\}$, shareholder i's payoff is

$$U_i(\mathrm{P}_j, x_j, \delta_i) = \mathrm{P}_j \times (x_j + \delta_i). \tag{2}$$

Payoffs are normalized to zero when the proposal fails. When the proposal passes, the payoff reflects two components: the common value x_j, which represents the proposal's impact on firm value, and the private value δ_i. Shareholders with higher δ_i are more willing to support management, as they earn higher utility when proposals succeed.

All dispersed shareholders share the same preference parameter δ_D, while each blockholder b has its own δ_b. The complete preference environment is thus given by $\Delta = \{\delta_D, \{\delta_b\}_{b=1}^{N_B}\}$. To facilitate interpretation, note that

$$\Pr(x_j + \delta_i > 0) = 1 - \Phi(-\delta_i) = \Phi(\delta_i), \tag{3}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function (CDF). This quantity gives the fraction of proposals that each shareholder believes should pass, or their baseline approval probability. We refer to $\Phi(\delta_i)$ as shareholder i's preference-implied support rate.

Ownership. Blockholders, indexed by $b = 1, \ldots, N_B$, each own a fraction ψ_b of outstanding shares, such that total blockholder ownership is $\Psi_B = \sum_{b=1}^{N_B} \psi_b$. The remaining fraction $\Psi_D = 1 - \Psi_B$ is held by N_D dispersed shareholders, each of whom owns a fraction $\psi_D = \Psi_D / N_D$. We focus on the asymptotic behavior of the model as $N_D \to \infty$, so that ownership converges to a finite set of blockholders and a continuum of dispersed shareholders. For analytical simplicity, ownership is identical across firms (ψ_b does not vary with j), we relax this restriction in the empirical implementation.

Voting Rule. Each firm holds a vote to decide whether the proposal passes or fails. We assume full turnout for analytical clarity, though when we estimate the model we allow for abstentions. The proposal passes, $\mathrm{P}_j = 1$, if the total support rate λ_j exceeds the passing threshold λ^*, and fails, $\mathrm{P}_j = 0$, otherwise. Under a majority rule, for example, $\lambda^* = 0.5$, whereas under a supermajority standard $\lambda^* > 0.5$.

Let $V_{bj} \in \{0, 1\}$ denote blockholder b's vote, and define the vector of blockholder votes $\mathbf{V}_{Bj} = [V_{1j}, \ldots, V_{N_B j}]$ and the vector of ownership weights $\mathbf{\Psi}_B = [\psi_1, \ldots, \psi_{N_B}]$. Given $\mathbf{V}_{Bj}$, the share of total votes cast in favor by blockholders is $s_{Bj} = \mathbf{V}_{Bj} \mathbf{\Psi}'_B$. Let τ_j denote the fraction of dispersed shareholders who vote for the proposal. The total support rate is therefore

$$\lambda_j = s_{Bj} + \tau_j = \mathbf{V}_{Bj} \mathbf{\Psi}'_B + \tau_j \Psi_D, \tag{4}$$

and the proposal passes if $\lambda_j \geq \lambda^*$.

2.2. Information Environment

Overview. Before voting, shareholders receive noisy signals about the proposal's underlying quality. Each shareholder updates her beliefs about the latent state x_j based on both a public signal shared by all investors and an idiosyncratic private signal. This information environment captures heterogeneity in how precisely different shareholders can assess proposal quality and provides the foundation for strategic voting in equilibrium.

Public signal. All shareholders observe a public signal that reflects common information about proposal quality,

$$\eta_j = x_j + u_j, \tag{5}$$

where the public noise term is distributed $u_j \sim N(0, \sigma_u^2)$ and is independent of x_j. The signal η_j can be interpreted as the information conveyed by publicly available reports or analyst coverage relevant to the proposal. The precision of the public signal, $1/\sigma_u^2$, determines how strongly it correlates with the true proposal quality. When σ_u^2 is small, the public signal is highly informative, and shareholders' posterior beliefs about x_j are tightly concentrated around η_j.

Private signals. In addition to the public signal, each shareholder i receives a private signal,

$$z_{ij} = x_j + \varepsilon_{ij}, \tag{6}$$

where $\varepsilon_{ij} \sim N(0, \sigma_{\varepsilon_i}^2)$ is independent across shareholders and independent of both x_j and u_j. This private signal represents investor-specific information or analysis that may not be reflected in the public signal. Differences in the precision of these private signals, $1/\sigma_{\varepsilon_i}^2$, capture heterogeneity in shareholders' information quality.

Dispersed shareholders share a common private-signal precision,

$$\sigma_{\varepsilon_i}^2 = \sigma_{\varepsilon_D}^2 \quad \text{for all dispersed } i, \tag{7}$$

reflecting that small investors rely on similar, relatively noisy sources of information. By contrast, each blockholder b may have a distinct information precision,

$$\sigma_{\varepsilon_i}^2 = \sigma_{\varepsilon_b}^2 \quad \text{for blockholder } b, \tag{8}$$

allowing blockholders to differ in how well-informed they are about proposal quality. This heterogeneity is central to the model, as it allows some blockholders to act on more accurate assessments of firm value than others.

Proxy advisor recommendation. In the data, we also observe the voting recommendation of Institutional Shareholder Services (ISS). We model ISS's recommendation as a deterministic function of the public signal:

$$\text{ISS}_j = \mathbb{1}\left[\eta_j > \xi\right], \tag{9}$$

where ξ is a fixed threshold. When the public signal exceeds this threshold, ISS recommends voting in favor of the proposal. This assumption reflects that ISS relies primarily on public information to form its guidance. The inclusion of ISS recommendations in the information structure allow us to identify σ_u^2 and estimate the informativeness of public information.

Interpretation. This structure implies that all shareholders share a common prior over proposal quality but differ in the precision of their private information. The public signal introduces correlation across their beliefs, while the ISS recommendation acts as a binary transformation of that signal. Together, these components determine how information aggregates through voting and how individual signals interact with the public component to shape equilibrium behavior.

2.3. Voting Strategy

Posterior beliefs. Given the information environment described above, each shareholder observes two signals about proposal quality: the public signal η_j and her private signal z_{ij}. Conditional on these signals, shareholder i forms a posterior belief about the latent state x_j

using standard Gaussian updating. Let

$$m_{ij} = \mathbb{E}[x_j \mid \eta_j, z_{ij}] \tag{10}$$

denote the posterior mean. Because of the normal–normal structure, m_{ij} is the sufficient statistic for the shareholder's voting decision.

Voting rule. Shareholder i votes in favor of the proposal if and only if her posterior expectation of proposal quality exceeds a cutoff k_i that reflects her preference parameter δ_i:

$$V_{ij} = 1 \quad \Longleftrightarrow \quad \mathbb{E}[x_j \mid \eta_j, z_{ij}] > k_i.$$

Equivalently, we can think of k_i as the shareholder's decision threshold: shareholders with lower k_i are more likely to support proposals, whereas those with higher k_i require stronger evidence of proposal quality to vote in favor.

Posterior mean. Because both signals are normally distributed, the posterior distribution of x_j given (η_j, z_{ij}) is normal with mean m_{ij} and variance $\sigma^2_{x|\eta,z}$. Specifically,

$$x_j \mid (\eta_j, z_{ij}) \sim N\big(m_{ij},\, \sigma^2_{x|\eta,z}\big),$$

where the posterior mean is

$$m_{ij} = \frac{\dfrac{\eta_j}{\sigma_u^2} + \dfrac{z_{ij}}{\sigma_{\varepsilon_i}^2}}{1 + \dfrac{1}{\sigma_u^2} + \dfrac{1}{\sigma_{\varepsilon_i}^2}}.$$

Conditional on the true proposal quality x_j and the public signal η_j, the only source of randomness is the idiosyncratic noise ε_{ij} in the private signal. Substituting $z_{ij} = x_j + \varepsilon_{ij}$ gives

$$m_{ij} = \frac{\dfrac{\eta_j}{\sigma_u^2} + \dfrac{x_j + \varepsilon_{ij}}{\sigma_{\varepsilon_i}^2}}{\Lambda_i} = \frac{\dfrac{\eta_j}{\sigma_u^2} + \dfrac{x_j}{\sigma_{\varepsilon_i}^2}}{\Lambda_i} + \frac{\varepsilon_{ij}}{\sigma_{\varepsilon_i}^2 \Lambda_i}, \tag{11}$$

where

$$\Lambda_i \equiv 1 + \frac{1}{\sigma_u^2} + \frac{1}{\sigma_{\varepsilon_i}^2}.$$

Because $\varepsilon_{ij} \sim N(0, \sigma_{\varepsilon_i}^2)$, it follows that

$$m_{ij} \mid (\eta_j, x_j) \sim N\left(\frac{\frac{\eta_j}{\sigma_u^2} + \frac{x_j}{\sigma_{\varepsilon_i}^2}}{\Lambda_i},\ \frac{1}{\sigma_{\varepsilon_i}^2 \Lambda_i^2} \right).$$

Voting probability. The probability that shareholder i supports the proposal, conditional on (x_j, η_j), is

$$\Pr(V_{ij} = 1 \mid \eta_j, x_j) = \Pr\left(m_{ij} > k_i \mid \eta_j, x_j\right) = \Phi\left(\sigma_{\varepsilon_i} \left[\frac{\eta_j}{\sigma_u^2} + \frac{x_j}{\sigma_{\varepsilon_i}^2} - k_i \Lambda_i \right] \right), \tag{12}$$

where $\Phi(\cdot)$ denotes the standard normal cumulative distribution function. This expression shows that a shareholder is more likely to vote for the proposal when either the public signal or her private signal is high, or when her cutoff k_i is low.

Equation (12) summarizes how each shareholder's decision depends on her information and preferences. The first two terms inside the brackets represent the information-based component of the decision: the public signal η_j and the true proposal quality x_j weighted by their respective precisions. The last term, $k_i\Lambda_i$, captures heterogeneity in voting thresholds across shareholders. A low threshold corresponds to a more pro-management stance or stronger preference for passing proposals. Heterogeneity in $(\sigma_{\varepsilon_i}^2, k_i)$ across shareholders thus generates cross-sectional variation in both voting behavior and responsiveness to information, which are key for identifying preferences in the empirical estimation.

2.4. Equilibrium

In this section, we lay out the components that determine the equilibrium and then define the equilibrium.

2.4.1. Equilibrium components

Pivotality. Each shareholder votes strategically, recognizing that her action affects payoffs only when it changes the voting outcome. In other words, although a shareholder does not know ex ante whether she will be pivotal, her vote is otherwise immaterial; hence she best responds as if at the pivotal margin and focuses on states where the outcome is close (Bond and Eraslan, 2010).

Formally, let the total support rate be $\lambda_j(v_i, V_{Bj}^{-i}, \tau_j) = \psi_i v_i + \Psi_{\text{for}}(V_{Bj}^{-i}) + \Psi_D \tau_j$, where $v_i \in \{0, 1\}$ is i's vote, ψ_i is her voting weight (for blockholder b, $\psi_i = \psi_b$; for a dispersed shareholder, $\psi_i = 0$ in the continuum limit), V_{Bj}^{-i} collects the other blockholders' votes, $\Psi_{\text{for}}(V_{Bj}^{-i}) \equiv \sum_{\ell \neq i} \psi_\ell V_{\ell j}$, and τ_j is the dispersed yes-share. The proposal passes iff $\lambda_j(\cdot) \geq \lambda^*$. Shareholder i is pivotal when her vote flips the outcome,

$$\text{piv}_{ij} \equiv \left\{\lambda_j(1, V_{Bj}^{-i}, \tau_j) \geq \lambda^*\right\} \cap \left\{\lambda_j(0, V_{Bj}^{-i}, \tau_j) < \lambda^*\right\}.$$

Thus, for a blockholder b with weight $\psi_b > 0$, pivotality occurs exactly when the dispersed support lies in the band

$$\tau_j \in \left[\frac{\lambda^* - \psi_b - \Psi_{\text{for}}(V_{Bj}^{-b})}{\Psi_D}, \frac{\lambda^* - \Psi_{\text{for}}(V_{Bj}^{-b})}{\Psi_D}\right),$$

whereas for a dispersed shareholder with $\psi_i = 0$ (continuum limit), pivotality reduces to the knife-edge

$$\tau_j = \frac{\lambda^* - \Psi_{\text{for}}(V_{Bj})}{\Psi_D}.$$

Best responses. When deciding how to vote, each shareholder compares the expected payoff from supporting the proposal to that from opposing it, recognizing that her vote only matters when she is pivotal. Let $m_{ij} = \mathrm{E}[x_j \mid \eta_j, z_{ij}]$ denote the posterior mean from the information environment and k_i denote the shareholder's cutoff determining their vote. Conditional on being pivotal and observing a posterior mean exactly at the cutoff, $m_{ij} = k_i$, the shareholder is indifferent between voting yes or no. Formally, the best-response condition is

$$\mathrm{E}\left[\, x_j \,\middle|\, \eta_j,\ m_{ij} = k_i,\ \mathrm{piv}_{ij} \,\right] = -\delta_i. \tag{13}$$

equivalently, this can be written as

$$\mathrm{E}\left[\, x_j \,\middle|\, \eta_j,\ z_{ij} = z^{\star}_{ij},\ \mathrm{piv}_{ij} \,\right] = -\delta_i. \tag{14}$$

where

$$z^{\star}_{ij}(\eta_j) = \sigma^2_{\varepsilon i}\left(\Lambda_i\, k_i - \frac{\eta_j}{\sigma^2_u}\right).$$

At the pivotal margin, the expected value of the proposal conditional on receiving the cutoff signal exactly offsets the shareholder's private preference δ_i. If the expected benefit is positive, she strictly prefers to vote in favor. Eq. (14) thus characterizes the equilibrium cutoff k_i that makes the shareholder just willing to support the proposal when her vote is decisive.

Signal and strategic effects. Expanding the expected payoff conditional on being pivotal yields

$$\mathrm{E}[x_j \mid \eta_j, z_{ij}, \mathrm{piv}_{ij}] = \int_x x_j\, f\left(x_j \mid z_{ij}, \eta_j, \mathrm{piv}_{ij}\right)\, dx,$$

where the posterior density of proposal quality is given by

$$f\left(x_j \mid \eta_j, z_{ij}, \mathrm{piv}_{ij}\right) \propto$$
$$f(x_j)\quad \underbrace{f(\eta_j \mid x_j)}_{\text{public signal effect}}\quad \underbrace{f\left(z_{ij} \mid x_j\right)}_{\text{private signal effect}}\quad \underbrace{\Pr\left(\mathrm{piv}_{ij} \mid x_j, \eta_j\right)}_{\text{strategic effect}} \tag{15}$$

We refer to the first two braced terms as the *signal effect*, as they capture the information about proposal quality revealed through the shareholder's own signals (η_j, z_{ij}). The final term represents the *strategic effect* as it captures the additional information conveyed by the event of being pivotal, which depends on how other shareholders vote. Conditioning on pivotality (weakly) shifts shareholders' beliefs toward values of x_j for which the overall vote is close, revealing information about proposal quality beyond what is contained in her own signals. An important point about the strategic effect is that it makes a particular shareholder's equilibrium voting strategy a function of other shareholders' equilibrium strategies, and hence a function of other shareholders' private information σ_{-i} and private value δ_{-i}.

2.4.2. Posterior beliefs about proposal quality

Determining equilibrium voting strategies requires evaluating the posterior in (15). This requires computing each shareholder's probability of being pivotal for each possible pair (x_j, η_j). Following Myatt (2015) and Zachariadis et al. (2020), we derive the limit of the game as N_D grows large. We give explicit expressions of (15) for dispersed shareholders and blockholders separately in the following proposition.

Proposition 1a: Dispersed shareholder posterior density

Symmetry in strategies, information, and preferences implies that each dispersed shareholder's best-response condition is the same, so there is a single limit to derive. As $N_D \to \infty$, the posterior density for any x_j converges as follows:

$$\lim_{N_D\to\infty} f\left(x_j \mid z_{Dj},\ \eta_j,\ \mathrm{piv}_{Dj}\right) \propto$$

$$f(x_j) \sum_{V_{Bj}} \underbrace{f\left(z_{Dj} \mid x_j\right)}_{\textit{signal effect}} \underbrace{\Pr\left(V_{Bj} \mid x_j, \eta_j\right)}_{\textit{strategic effect}} \underbrace{\mathrm{dirac}\left(p_D(x_j,\eta_j) - \tau^{V_{Bj}}\right)}_{\textit{pivotal constraint}} = \tag{16a}$$

$$\sum_{V_{Bj}} \underbrace{f\left(x_{\eta_j}^{V_{Bj}}\right)}_{\textit{prior}} \underbrace{f\left(z_{Dj} \mid x_{\eta_j}^{V_{Bj}}\right)}_{\textit{signal effect}} \underbrace{\chi^{V_{Bj}}}_{\textit{Jacobian}} \underbrace{\Pr\left(V_{Bj} \mid x_{\eta_j}^{V_{Bj}}, \eta_j\right)}_{\textit{strategic effect}} \mathrm{dirac}\left(x_j - x_{\eta_j}^{V_{Bj}}\right). \tag{16b}$$

The following objects are required:

- *Dispersed voting probability:*

$$p_D(x_j,\eta_j) = \Phi\left(\sigma_D\left[\frac{\eta_j}{\sigma_u^2} + \frac{x_j}{\sigma_D^2} - k_D\Lambda_D\right]\right), \qquad \Lambda_D = 1 + \frac{1}{\sigma_u^2} + \frac{1}{\sigma_D^2},$$

- *Value of x_j when a dispersed shareholder is pivotal, given profile V_{Bj} (i.e., the solution to*

$p_D(x_j, \eta_j) = \tau^V_{B_j}$):

$$x_{\eta_j}^{V_{Bj}} = \sigma_D^2 \left(\Lambda_D k_D - \frac{\eta_j}{\sigma_u^2} + \frac{q^{V_{Bj}}}{\sigma_D} \right), \qquad q^{V_{Bj}} \equiv \Phi^{-1}\big(\tau^{V_{Bj}}\big),$$

- *Jacobian correction term* $\operatorname{dirac}\big(p_D(x_j, \eta_j) - \tau^{V_{Bj}}\big) \mapsto x_j = x_{\eta_j}^{V_{Bj}}$:

$$\chi^{V_{Bj}} = \frac{\sigma_D}{\phi\big(q^{V_{Bj}}\big)},$$

- *Threshold dispersed share:*

$$\tau^{V_{Bj}} = \frac{\lambda^* - \Psi_{for}(V_{Bj})}{\Psi_D},$$

- *Blockholder vote profile likelihood given* (x_j, η_j)*:*

$$\Pr\left(V_{Bj} \mid x_j, \eta_j\right) = \prod_{b=1}^{N_B} \left[\Phi\big(A_b \eta_j + B_b x_j - C_b\big)\right]^{V_{Bj}(b)} \left[1 - \Phi\big(A_b \eta_j + B_b x_j - C_b\big)\right]^{1-V_{Bj}(b)},$$

where

$$A_b = \frac{\sigma_b}{\sigma_u^2}, \qquad B_b = \frac{1}{\sigma_b}, \qquad C_b = \sigma_b \Lambda_b k_b, \qquad \Lambda_b = 1 + \frac{1}{\sigma_u^2} + \frac{1}{\sigma_b^2}.$$

Remark on Proposition 1a. The Dirac delta function in (16a) enforces the pivotality condition for dispersed shareholders by restricting the posterior density to values of the latent proposal quality x_j that make a dispersed voter exactly pivotal: those for which the implied dispersed support rate $p_D(x_j, \eta_j)$ equals the threshold share $\tau^{V_{Bj}}$ consistent with a particular blockholder voting profile V_{Bj}. Intuitively, as $N_D \to \infty$, there is a unique draw of x_j (conditional on η_j) for which a dispersed shareholder is pivotal, and the posterior density tilts strongly towards these states. Each such outcome corresponds to one blockholder vote profile and its associated knife-edge quality $x_{\eta_j}^{V_{Bj}}$, with its contribution weighted by the prior density $f(x_{\eta_j}^{V_{Bj}})$, the likelihood of the dispersed signal $f(z_{Dj} \mid x_{\eta_j}^{V_{Bj}})$, the Jacobian correction term $\chi^{V_{Bj}}$, and the strategic likelihood $\Pr\left(V_{Bj} \mid x_{\eta_j}^{V_{Bj}}, \eta_j\right)$.

Proposition 1b: Blockholder posterior density

Given differences in preferences and information, each blockholder will have a separate posterior density of the state, which will be a function of the dispersed shareholder support τ_j *and the* *blockholder voting profile excluding b, $V_{B,-b,j}$. As $N_D \to \infty$:*

$$\lim_{N_D\to\infty} f\left(x_j \mid z_{bj}, \eta_j, \mathrm{piv}_{bj}\right) \propto$$
$$f(x_j)\underbrace{f\left(z_{bj} \mid x_j\right)}_{\textit{signal effect}} \sum_{V_{B,-b,j}} \underbrace{\mathbb{1}\left[p_D(x_j,\eta_j) \in \left[\tau_L^{V_{B,-b,j}},\ \tau_H^{V_{B,-b,j}}\right]\right]}_{\textit{pivotality}} \underbrace{\Pr\left(V_{B,-b,j} \mid x_j, \eta_j\right)}_{\textit{strategic effect}}. \tag{17}$$

The following objects are required:

- *Pivotal-bounding proposal qualities given η_j and $V_{B,-b,j}$:*

$$x_{L,\eta_j}^{V_{B,-b,j}} = \sigma_D^2\left(\Lambda_D k_D - \frac{\eta_j}{\sigma_u^2} + \frac{q_L^{V_{B,-b,j}}}{\sigma_D}\right), \qquad x_{H,\eta_j}^{V_{B,-b,j}} = \sigma_D^2\left(\Lambda_D k_D - \frac{\eta_j}{\sigma_u^2} + \frac{q_H^{V_{B,-b,j}}}{\sigma_D}\right),$$

  with $q_L^{V_{B,-b,j}} = \Phi^{-1}\left(\tau_L^{V_{B,-b,j}}\right)$ and $q_H^{V_{B,-b,j}} = \Phi^{-1}\left(\tau_H^{V_{B,-b,j}}\right)$.

- *Pivotal-bounding dispersed support rates given $V_{B,-b,j}$:*

$$\tau_L^{V_{B,-b,j}} = \frac{\lambda^* - \Psi_{for}(V_{B,-b,j}) - \psi_b}{\Psi_D}, \qquad \tau_H^{V_{B,-b,j}} = \frac{\lambda^* - \Psi_{for}(V_{B,-b,j})}{\Psi_D}.$$

- *Blockholder voting profile (excluding b) likelihood, conditional on (x_j, η_j):*

$$\Pr\left(V_{B,-b,j} \mid x_j, \eta_j\right) = \prod_{\substack{b'=1\\ b'\neq b}}^{N_B} \left[\Phi\left(A_{b'}\eta_j + B_{b'}x_j - C_{b'}\right)\right]^{V_{B,-b,j}(b')} \left[1-\Phi\left(A_{b'}\eta_j + B_{b'}x_j - C_{b'}\right)\right]^{1-V_{B,-b,j}(b')},$$

  where

$$A_{b'} = \frac{\sigma_{b'}}{\sigma_u^2}, \qquad B_{b'} = \frac{1}{\sigma_{b'}}, \qquad C_{b'} = \sigma_{b'}\Lambda_{b'}k_{b'}, \qquad \Lambda_{b'} = 1 + \frac{1}{\sigma_u^2} + \frac{1}{\sigma_{b'}^2}.$$

Remark on Proposition 1b. Given the voting profile of other blockholders $\mathbf{V}_{B_j}^{-b}$, the fraction of shares cast in favor by all shareholders excluding b is $s_B^{-b} + \tau_j\Psi_D$. Blockholder b is pivotal when this lies in $[\lambda^* - \psi_b,\ \lambda^*]$, which is equivalent to the dispersed support rate falling in $[\tau_L^{V_{B,-b,j}},\ \tau_H^{V_{B,-b,j}}]$, i.e., when η_j lies in the corresponding window. The blockholder scales her prior by the likelihood of observing her private signal z_{bj} (the signal effect), and a summation that reflects the strategic effect. In each summand, the probability of observing blockholder votes $\mathbf{V}_{B_j}^{-b}$ is scaled by the probability of observing a dispersed support rate within the given bound τ_L and τ_H. In the case of a single blockholder, the summation collapses to a single integral, because the pivot probability is determined only by the dispersed shareholders' votes

2.4.3. Equilibrium Definition

The equilibrium concept is pure strategy Bayesian–Nash, in which each shareholder chooses a cutoff strategy that maximizes expected utility given her information set and beliefs about pivotality. Let the dispersed and block shareholders observe a common public signal $\eta_j = x_j + u_j$ and private signals $z_{ij} = x_j + \varepsilon_{ij}$, with (u_j, ε_{ij}) jointly independent and mean-zero Gaussian. Each shareholder votes "For" if the posterior mean of proposal quality exceeds her individual cutoff:

$$\mathrm{E}\left[x_j \mid \eta_j, z_{Dj}, \mathrm{piv}_{Dj}\right] + \delta_D \geq 0 \iff m_{Dj} \geq k_D, \tag{18}$$

$$\mathrm{E}\left[x_j \mid \eta_j, z_{bj}, \mathrm{piv}_{bj}\right] + \delta_b \geq 0 \iff m_{bj} \geq k_b, \quad \forall b \in \{1, \dots, N_B\}, \tag{19}$$

where $m_{ij} = \mathrm{E}[x_j \mid \eta_j, z_{ij}]$ denotes shareholder i's posterior mean under Gaussian updating.

Equilibrium Support Rates. The equilibrium can be equivalently written as a system of indifference conditions:

$$\begin{aligned} \mathrm{E}[x_j \mid \eta_j,\, m_{Dj} = k_D,\, \mathrm{piv}_{Dj}] &= -\,\delta_D, \\ \mathrm{E}[x_j \mid \eta_j,\, m_{bj} = k_b,\, \mathrm{piv}_{bj}] &= -\,\delta_b, \quad \forall b \in \{1, \dots, N_B\}. \end{aligned}$$

Each cutoff k_i reflects the shareholder's strategic inference under the event of being pivotal, which in turn depends on the aggregate support rate implied by others' strategies.

Given an equilibrium cutoff profile $\mathbf{k} = (k_D, k_1, \dots, k_{N_B})$, the (prior) probability that a shareholder votes "For", their *equilibrium support rate*, is

$$\Pr\left(V_{ij} = 1\right) = \Pr\left(m_{ij} \geq k_i\right) = \iint 1\{m_{ij}(\eta, z_i) \geq k_i\} f(\eta)\, f(z_i \mid \eta)\, d\eta\, dz_i, \tag{20}$$

where $f(\eta)$ and $f(z_i \mid \eta)$ denote the public- and private-signal densities implied by $(\sigma_u^2, \sigma_{\varepsilon_i}^2)$.

3. Model Analysis

In this section, we undertake numerical analysis of the model to understand how information and preferences interact in our voting game.

3.1. Preference and Support Rate

A common empirical interpretation of proxy voting data is that investors who support management proposals more frequently are more pro-management. This reasoning treats voting support rates as direct measures of preference. However, in our model, voting behavior arises endogenously from strategic and informational considerations. Each shareholder votes as if pivotal by comparing the expected value of the proposal conditional on being decisive. This conditioning can fundamentally changes the relationship between a shareholder's latent preference and her observed support rate.

Intuitively, a shareholder with a stronger pro-management preference δ_i is willing to approve weaker proposals, but doing so alters the states in which she is pivotal. When she becomes more lenient, her vote is no longer decisive in high-quality proposals, which would pass regardless, and becomes pivotal primarily in marginal or lower-quality cases. As a result, the distribution of proposals in which her vote matters shifts toward states with lower average quality x_j. This reweighting muddles the relation between preferences and votes: even though a shareholder may prefer approval, her *equilibrium* probability of voting "For" can fall.

To illustrate this mechanism in the simplest possible setting, consider a two-shareholder version of the model in which the public signal is uninformative, i.e., $\sigma_u \to \infty$. In this limit, each shareholder votes solely based on her private signal, and the pivotality logic becomes transparent. We further simplify by assuming that one shareholder, the dispersed shareholder D, votes mechanically according to a fixed rule rather than responding strategically to her pivotal probability. The other shareholder, a blockholder b, behaves strategically, choosing her vote optimally based on the expected payoff conditional on being pivotal. This assumption isolates the mechanism of interest by removing strategic feedback between the two players: the blockholder internalizes how her preference affects the pivotal states, while the dispersed shareholder's behavior is exogenous.

Let the two shareholders together own the entire firm, with ownership shares $\psi_b = 0.6, \Psi_D = 0.4$. Each shareholder $i \in \{b, D\}$ observes a private signal about the true proposal quality x, $z_i = x + \varepsilon_i$,, with $\varepsilon_i \sim N(0, \sigma_i^2)$, and votes *for* the proposal whenever the posterior mean exceeds her cutoff k_i. Since the prior is standard normal, this rule is equivalent to voting in favor whenever $z_i > k_i$, so a higher cutoff represents a stricter voting stance.

Pivotality and equilibrium intuition. Each shareholder is pivotal when her vote changes the outcome of the proposal. The dispersed shareholder is pivotal only if the blockholder votes against the proposal, since her 0.4 ownership just pushes total support above $\lambda^* = 0.5$. Conversely, the blockholder is pivotal when the dispersed shareholder votes against it, because her 0.6 ownership is just enough to make the proposal pass. Thus, pivotality arises only in borderline cases when the other shareholder's vote leaves the total support rate just below the passing threshold.

Lenient cutoffs. Suppose both shareholders start with lenient cutoffs $k_b = k_D = 0$, meaning they vote *For* whenever their signal is positive. In this case, when the true proposal quality x is high, both signals are likely positive and both vote *For*, so the proposal passes easily. When x is low, both vote *Against*, and the proposal fails easily. Each shareholder is pivotal only when x is close to zero—precisely where her signal z_i is near the cutoff k_i. Hence, conditional on being pivotal, $\mathrm{E}[x \mid z_i = k_i, \mathrm{piv}_i]$ is close to zero.

Increasing the blockholder's cutoff. Now let the blockholder become stricter, raising her cutoff to $k_b = 1$. She now requires stronger evidence in favor of the proposal to vote *For*. When x is high (say $x = +1.5$), her signal is likely well above the cutoff, so she votes *For* and the proposal passes comfortably, so she is not pivotal, similarly for low x, e.g. $x = -1.5$. Only when x is moderate or slightly negative (around $x = 0$) does her vote potentially decide the outcome, since the dispersed shareholder's 0.4 weight alone is insufficient to make the proposal pass. Therefore, as the blockholder raises her cutoff, the range of states (x, z_b) in which she is pivotal shifts toward lower-quality proposals. Conditional on being pivotal, the expected proposal quality decreases:

$$\frac{\partial}{\partial k_i}\mathrm{E}[x \mid z_i = k_i, \mathrm{piv}_i] < 0.$$

Equilibrium indifference and preference. In equilibrium, each shareholder's cutoff k_i satisfies the indifference condition

$$\mathrm{E}[x \mid z_i = k_i,\ \mathrm{piv}_i] = -\delta_i,$$

where δ_i is the shareholder's preference: higher δ_i corresponds to being more pro-management. Because $\mathrm{E}[x \mid z_i = k_i, \mathrm{piv}_i]$ is decreasing in k_i, a larger preference δ_i (right-hand side lower) requires a larger k_i to restore equality. Hence, more preferenceed shareholders choose high cutoffs: they vote *For* more easily and are willing to support weaker proposals.

Interpretation. This example highlights the equilibrium interaction between preference, information, and pivotality. As a shareholder becomes stricter (higher k_i), she ceases to be pivotal in high-quality proposals that would pass regardless. The only proposals for which her vote matters become more marginal. Thus, conditional on being pivotal, she expects the proposal's true quality to be lower on average. Formally, since $\mathrm{E}[x \mid z_i = k_i, \mathrm{piv}_i]$ declines with k_i, the equilibrium indifference condition implies the cutoff k_i increases in δ_i.

Consequently, in the two-shareholder world, a higher preference δ_i can correspond to a higher equilibrium cutoff and a lower observed support rate. This illustrates a central insight of our analysis: *in a strategic voting equilibrium, higher observed support rates need not indicate stronger managerial alignment.*

3.2. Numerical Examples

3.2.1. Background

To illustrate the economics of the model, we present a series of simple numerical examples that highlight how pivotality and information interact in shareholders' voting behavior. Through-out these examples, we fix the preference and ownership parameters at

$$\delta_D = \delta_b = 0, \qquad \Psi_D = 0.8, \qquad \psi_b = 0.2,$$

which together imply equilibrium cutoffs $k_D = k_b = 0$ for both the dispersed shareholder and the blockholder. We set the passing threshold at $\lambda^* = 0.5$, so that the blockholder becomes pivotal only when the dispersed shareholders contribute 30% to 50% votes for management, which implies that the support rate of dispersed shareholders are between 0.375 and 0.625. Unless otherwise noted, both shareholders receive neutral private signals,

$$z_D = 0, \qquad z_b = 0,$$

so that their posterior beliefs depend entirely on the realization of the public signal η and on the information implied by pivotality.

Pivotal window and posterior updates. The blockholder is pivotal only when the dispersed shareholders' support lies in

$$\tau_D \in (\tau_L, \tau_H), \qquad \tau_L = \frac{\lambda^* - \psi_b}{\Psi_D}, \qquad \tau_H = \frac{\lambda^*}{\Psi_D}.$$

Under the posterior–mean cutoff rule, a dispersed shareholder votes *For* if

$$p_D(x, \eta) = \Phi\!\left(\sigma_D\left[\frac{\eta}{\sigma_u^2} + \frac{x}{\sigma_D^2} - \Lambda_D k_D\right]\right), \qquad \Lambda_D = 1 + \frac{1}{\sigma_u^2} + \frac{1}{\sigma_D^2}.$$

Pivotality occurs when $p_D(x, \eta)$ equals either boundary τ_L or τ_H. Let $q_{L/H} \equiv \Phi^{-1}(\tau_{L/H})$; then solving $p_D(x, \eta) = \tau_{L/H}$ for x gives

$$\sigma_D\left[\frac{\eta}{\sigma_u^2} + \frac{x}{\sigma_D^2} - \Lambda_D k_D\right] = q_{L/H},$$

which yields the pivotal quality window

$$x_{L/H}(\eta) = \sigma_D^2 \left(\Lambda_D k_D - \frac{\eta}{\sigma_u^2} + \frac{q_{L/H}}{\sigma_D} \right).$$

This range $[x_L(\eta), x_H(\eta)]$ identifies the proposal qualities consistent with the blockholder being decisive given the realized public signal η.

Posterior updates. Combining the prior $x \sim N(0,1)$ with the two signals (η, z_b) gives the unconstrained posterior

$$m_b = \frac{\frac{\eta}{\sigma_u^2} + \frac{z_b}{\sigma_b^2}}{\Lambda_b}, \qquad \sigma_{b,\text{post}}^2 = \frac{1}{\Lambda_b}, \qquad \Lambda_b = 1 + \frac{1}{\sigma_u^2} + \frac{1}{\sigma_b^2}.$$

Conditioning on pivotality restricts x to $[x_L(\eta), x_H(\eta)]$, giving a truncated normal posterior with mean

$$\mathbb{E}\left[x \mid \eta, z_b, \text{piv}_b\right] = m_b + \sigma_{b,\text{post}} \frac{\phi(\alpha) - \phi(\beta)}{\Phi(\beta) - \Phi(\alpha)}, \qquad \alpha = \frac{x_L(\eta) - m_b}{\sigma_{b,\text{post}}}, \quad \beta = \frac{x_H(\eta) - m_b}{\sigma_{b,\text{post}}}.$$

Comparing $\mathbb{E}[x \mid \eta, z_b, \text{piv}_b]$ to m_b isolates the *strategic effect*: conditioning on being pivotal alters the blockholder's belief about proposal quality relative to the baseline posterior.

3.2.2. Examples

We present a series of numerical examples to convey how changing information structures changes the impact of pivotality on blockholder beliefs. Table 1 summarizes the numerical examples.

Example 1: Neutral public signal. In the first example, we assume that $\sigma_u = \sigma_{\varepsilon b} = \sigma_{\varepsilon D} = 5$. Both shareholders observe a neutral realization $\eta = 0$. The blockholder's pivotality restricts proposal quality to a symmetric window

$$x \in [x_L(\eta), x_H(\eta)] = \left[-1.593,\ 1.593\right].$$

Because this window is centered around zero when $\eta = 0$, conditioning on being pivotal does not tilt beliefs: the pivotal posterior mean essentially coincides with the unconstrained posterior mean,

$$\mathbb{E}\left[x \mid \eta, \mathrm{piv}_b\right] = 0.$$

In this example, with a neutral public signal and moderate public noise, the pivotality filter is symmetric and conveys no directional information; strategic conditioning leaves the blockholder's expected quality unchanged. Small departures from $\eta = 0$ would break this symmetry and generate a nonzero strategic update.

Example 2: Mildly favorable public signal. We hold the same information structure $\sigma_u = \sigma_{\varepsilon b} = \sigma_{\varepsilon D} = 5$ and consider a mildly positive public signal $\eta = 1$. The blockholder's pivotality maps into an *asymmetric* quality window

$$x \in [x_L(\eta), x_H(\eta)] \;=\; \left[-2.593,\; 0.593\right].$$

Because $\eta > 0$ makes dispersed shareholders more likely to vote "*For*," staying near the passage threshold requires a *lower* latent quality x to offset the favorable public signal; hence, the pivotal window shifts to the left relative to the neutral-η case.

The unconstrained posterior mean (ignoring pivotality) is slightly positive,

$$m_b \;=\; \mathbb{E}[x \mid \eta, z_b] \;\approx\; 0.037,$$

reflecting the modestly favorable η. Conditioning on being pivotal truncates the posterior to $[x_L, x_H]$ and yields a substantially *lower* pivotal posterior mean,

$$\mathbb{E}[x \mid \eta, z_b, \mathrm{piv}_b] \;\approx\; -0.404,$$

a negative shift of about -0.44.

Economic intuition. When the public signal is favorable, being pivotal means the proposal is only barely passing despite $\eta > 0$. This can happen only if dispersed shareholders—who collectively determine most of the vote weight—received *unfavorable* private signals, pulling their support just low enough for the blockholder to matter. Observing that she is pivotal therefore reveals that others' private information must be negative, which in turn implies a lower expected proposal quality. In other words, being pivotal conveys bad news, leading the blockholder to revise her belief downward even though the public signal was favorable.

Example 3: Mildly unfavorable public signal. With the same information parameters and a mildly negative realization $\eta = -1$, the blockholder's pivotality implies an *asymmetric* quality window

$$x \in [x_L(\eta), x_H(\eta)] \;=\; \big[-0.593,\; 2.593\big].$$

Because $\eta < 0$ reduces dispersed support, staying near the passage threshold now requires a *higher* latent quality x to offset the adverse public signal; the pivotal window shifts to the right relative to the neutral-η case.

Ignoring pivotality, the posterior mean is slightly negative,

$$m_b \;=\; \mathbb{E}[x \mid \eta, z_b] \;\approx\; -0.037,$$

reflecting the unfavorable η. Conditioning on being pivotal truncates the posterior to $[x_L, x_H]$ and flips the inference,

$$\mathbb{E}[x \mid \eta, z_b, \mathrm{piv}_b] \;\approx\; 0.404,$$

a positive shift of about +0.44, mirroring the result in the Example 2

Economic intuition. When the public signal is unfavorable, the only way the proposal can still be at the margin of passing is if dispersed shareholders collectively received *positive* private signals, lifting support toward the threshold. The blockholder, upon realizing she is pivotal, interprets this as good news about the proposal's fundamentals: pivotality reveals that the dispersed crowd's information must be stronger than what the public signal alone suggests. As a result, her pivotal posterior is higher than her unconditional posterior. This case is the mirror image of Example 2 and demonstrates how pivotality can overturn the directional message of the public signal by revealing information about others' private beliefs.

The previous examples illustrated how, holding the information structure fixed, the *realization* of the public signal η changes the informational content of being pivotal. We now turn to how the *precision* of information—both public and private—shapes the magnitude of this strategic effect. When public information becomes more precise (lower σ_u), the public signal dominates individual beliefs, leaving less room for inference from others' actions. Conversely, when public information is noisy (higher σ_u), pivotality conveys more about others' private signals and thus exerts a stronger influence on posterior beliefs. Similarly, when dispersed shareholders' private information becomes more precise (lower σ_D), their voting decisions become more responsive to true proposal quality, amplifying the informational content of being pivotal for the blockholder.

In the next two numerical examples, we vary (σ_u, σ_D) while holding other parameters fixed to show how the precision of public and private signals governs the extent to which pivotality alters the blockholder's posterior belief and voting decision.

Example 4: Highly precise public information. We now reduce the noise in the public signal to $\sigma_u = 1$, keeping all other parameters fixed at $(\sigma_D, \sigma_b) = (5, 5)$ and $(\Psi_D, \psi_b) = (0.8, 0.2)$. The realization of the public signal is $\eta = 1$ while the realization of the blockholder's private signal is $z_b = 5$. Given the high precision of the public signal, dispersed shareholders' votes are largely synchronized with η, so that the proposal outcome is almost perfectly predictable from public information alone.

The blockholder's pivotal window is extremely narrow and located far in the left tail of the quality distribution,

$$x \in [x_L(\eta), x_H(\eta)] = \big[-26.59, -23.41\big].$$

This means that being pivotal can occur only in rare, very low-quality states where the favorable public signal has to be offset by exceptionally negative dispersed signals. Because the posterior distribution of x conditional on (η, z_b) already places negligible probability mass in this region, conditioning on pivotality adds almost no new information. Indeed, both the unconstrained and pivotal posteriors coincide, despite the private signal is strong

$$m_b \;=\; \mathbb{E}[x \mid \eta, z_b] \;=\; 0.588, \qquad \mathbb{E}[x \mid \eta, z_b, \mathrm{piv}_b] \;=\; 0.588.$$

Economic intuition. When public information is highly precise, the blockholder already infers nearly everything about proposal quality from η and her own signal. In such cases, pivotality is extremely unlikely and carries little additional informational content—it merely confirms what is already known. Thus, as public signals become more informative, the *strategic effect* of pivotality on beliefs effectively disappears.

Example 5: Precise blockholder private information. We now make the blockholder's private signal extremely precise, setting $\sigma_b = 0.1$ while keeping $\sigma_u = \sigma_D = 5$. The realized signals are $(z_D, z_b, \eta) = (0, 0, 5)$. The public signal is very favorable but imprecise, whereas the blockholder's private information is *highly precise* and happens to be neutral.

As before, the dispersed-side pivotality band $\tau_D \in (\tau_L, \tau_H) = (0.375, 0.625)$ maps, via the posterior–mean cutoff rule with $k_D = 0$, into the left–tail quality window

$$x \in [x_L(\eta), x_H(\eta)] \;=\; \big[-6.593,\; -3.407\big],$$

i.e., states where very low underlying quality offsets the favorable public signal so that the outcome remains knife–edge.

The blockholder's posterior mean places precision weights on η and z_b. With $\sigma_u = 5$ and $\sigma_b = 0.1$, the weight on z_b is $\frac{100}{101.04} \approx 0.990$ and the weight on η is $\frac{0.04}{101.04} \approx 0.000396$ Thus, even with $\eta = 5$ and $z_b = 0$,

$$m_b = \frac{5/25 + 0}{101.04} = 0.$$

Conditioning on pivotality truncates the posterior to $[x_L, x_H] = [-6.593, -3.407]$. But, because the blockholder's posterior is extremely concentrated around 0 with variance $1/\Lambda_b \approx 0.0099$, this truncation has essentially zero overlap with her posterior mass. Consequently,

$$\mathbb{E}[x \mid \eta, z_b, \mathrm{piv}_b] \;\approx\; m_b \;\approx\; 0,$$

Table 1. Numerical Examples: Information Structure, Signals, and Posterior Beliefs

	Ex. 1 Neutral η	Ex. 2 Mildly positive η	Ex. 3 Mildly negative η	Ex. 4 Precise public signal	Ex. 5 Precise private signal
Ownership (dispersed) Ψ_D	0.8	0.8	0.8	0.8	0.8
Ownership (blockholder) ψ_b	0.2	0.2	0.2	0.2	0.2
Public noise σ_u	5.0	5.0	5.0	1.0	5.0
Dispersed noise σ_D	5.0	5.0	5.0	5.0	5.0
Blockholder noise σ_b	5.0	5.0	5.0	5.0	0.1
Bias (dispersed) δ_D	0.0	0.0	0.0	0.0	0.0
Bias (blockholder) δ_b	0.0	0.0	0.0	0.0	0.0
Public signal η	0	1	−1	1	5
Private signal (dispersed) z_D	0	0	0	0	0
Private signal (blockholder) z_b	0	0	0	5	0
Lower pivotal bound $x_L(\eta)$	−1.59	−2.59	−0.59	−26.59	−6.59
Upper pivotal bound $x_H(\eta)$	1.59	0.59	2.59	−23.41	−3.41
Posterior (no pivotality) m_b	0.000	0.037	−0.037	0.588	0.002
Posterior (pivotal) $\mathbb{E}[x \mid \eta, z_b, \text{piv}_b]$	0.000	−0.404	0.404	0.588	0.002
Strategic shift $\Delta = \text{piv} - m_b$	0.000	−0.441	+0.441	0.000	0.000

so the pivotal posterior mean is (numerically) identical to the unconstrained mean.

Economic intuition. When the blockholder's private information is sufficiently precise, her belief is dominated by z_b and becomes sharply concentrated; the event "being pivotal"—which requires the world to lie in a far left–tail quality band implied by the dispersed crowd—carries (under her private posterior) vanishing probability mass. In this limit, conditioning on pivotality adds no information: the *strategic effect* of pivotality disappears as $\sigma_b \to 0$, even when the public signal is very favorable.

3.3. Two-shareholder Equilibrium

The numerical examples above illustrate how pivotality and information precision jointly shape blockholders' strategic updates. In particular, conditioning on being pivotal can substantially alter a shareholder's posterior belief about proposal quality, depending on the informativeness of public and private signals. We now turn to equilibrium analysis to further explore how these mechanisms interact with heterogeneity in preference. Specifically, we solve for the two–shareholder equilibria under different preference parameters, allowing the blockholder to be weakly more pro–management than the dispersed shareholder. Building on the economic intuition established in the numerical examples, this exercise clarifies the key identification question of the model: *can one infer a shareholder's underlying preference directly* *from her observed support rate?* The answer, as we show below, is generally no: support rates confound preference preference with strategic and informational effects.

Equilibrium Definition. An equilibrium is a pair (k_D, k_b) such that the pivotal conditional expectations equal the negative preferencees:

$$-\delta_D = E_D[x_j|\eta_j, z_D = k_D; k_b], \tag{21}$$

$$-\delta_b = E_b[x_j|\eta_j, z_b = k_D; k_D]. \tag{22}$$

Dispersed Pivotal Moment $E_D[x_j|\eta_j, z_D = k_D; k_b]$. Let the dispersed pivotal constraint be that the dispersed *For* share equals the threshold implied by the blockholder's vote. With one blockholder, the two blockholder states are $v \in \{0, 1\}$ (vote *Against*/*For*). Define the dispersed threshold shares

$$\tau^{(0)} \equiv \frac{\lambda^*}{\Psi_D}, \qquad \tau^{(1)} \equiv \frac{\lambda^* - \psi_b}{\Psi_D},$$

clamped to $(0, 1)$, and set $q^{(v)} \equiv \Phi^{-1}(\tau^{(v)})$. Solving $p_D(x, \eta) = \tau^{(v)}$ for x yields the x-knife-edge:

$$x_\eta^{(v)} = \sigma_D^2\left(\Lambda_D k_D - \frac{\eta}{\sigma_u^2} + \frac{q^{(v)}}{\sigma_D}\right), \qquad \Lambda_D \equiv 1 + \frac{1}{\sigma_u^2} + \frac{1}{\sigma_D^2}. \tag{23}$$

The Jacobian from $p_D \mapsto x$ at the knife-edge is

$$\chi^{(v)} = \frac{\sigma_D}{\phi\big(q^{(v)}\big)}. \tag{24}$$

The knife-edge private signal for the dispersed investor is $z_D^*(\eta) = \sigma_D^2\Lambda_D k_D - (\sigma_D^2/\sigma_u^2)\eta$. Then the dispersed pivotal posterior expectation is

$$E_D[x_j|\eta_j, z_D = k_D; k_b] = \frac{\displaystyle\sum_{v\in\{0,1\}}\int_{-\infty}^{\infty} x_\eta^{(v)}\ \underbrace{f\big(x_\eta^{(v)}\big)}_{N(0,1)}\ \underbrace{f\big(\eta \mid x_\eta^{(v)}\big)}_{N(\eta;x_\eta^{(v)},\sigma_u^2)}\ \underbrace{f\big(z_D^*(\eta) \mid x_\eta^{(v)}\big)}_{N(z_D^*(\eta);x_\eta^{(v)},\sigma_D^2)}\ \chi^{(v)}\ \underbrace{\Pr\big(V_b = v \mid x_\eta^{(v)}, \eta\big)}_{\text{strategic}}\ d\eta}{\displaystyle\sum_{v\in\{0,1\}}\int_{-\infty}^{\infty} \underbrace{f\big(x_\eta^{(v)}\big)}_{N(0,1)}\ \underbrace{f\big(\eta \mid x_\eta^{(v)}\big)}_{N(\eta;x_\eta^{(v)},\sigma_u^2)}\ \underbrace{f\big(z_D^*(\eta) \mid x_\eta^{(v)}\big)}_{N(z_D^*(\eta);x_\eta^{(v)},\sigma_D^2)}\ \chi^{(v)}\ \underbrace{\Pr\big(V_b = v \mid x_\eta^{(v)}, \eta\big)}_{\text{strategic}}\ d\eta}. \tag{25}$$

Blockholder Pivotal Moment $E_b[x_j|\eta_j, z_b = k_D; k_D]$**.** The dispersed pivotal band for fixed η is

$$\tau_L \equiv \frac{\lambda^* - \psi_b}{\Psi_D}, \qquad \tau_H \equiv \frac{\lambda^*}{\Psi_D}, \qquad q_L = \Phi^{-1}(\tau_L), \quad q_H = \Phi^{-1}(\tau_H).$$

The corresponding x-window solves $p_D(x, \eta) = \tau_{\bullet}$:

$$x_L(\eta) = \sigma_D^2 \left(\Lambda_D k_D - \frac{\eta}{\sigma_u^2} + \frac{q_L}{\sigma_D} \right), \qquad x_H(\eta) = \sigma_D^2 \left(\Lambda_D k_D - \frac{\eta}{\sigma_u^2} + \frac{q_H}{\sigma_D} \right). \tag{26}$$

The blockholder's knife-edge private signal is $z_b^*(\eta) = \sigma_b^2 \Lambda_b k_b - (\sigma_b^2/\sigma_u^2)\eta$ with $\Lambda_b = 1 + \sigma_u^{-2} + \sigma_b^{-2}$. Let $f(z_b^*(\eta)|x) = N(z_b^*(\eta); x, \sigma_b^2)$. Then

$$E_b[x_j|\eta_j, z_b = k_D; k_D] = \frac{\displaystyle\int_{-\infty}^{\infty} \int_{x_L(\eta)}^{x_H(\eta)} x \underbrace{f(x)}_{N(0,1)} \underbrace{f(\eta \mid x)}_{N(\eta; x, \sigma_u^2)} \underbrace{f\big(z_b^*(\eta) \mid x\big)}_{N(z_b^*(\eta); x, \sigma_b^2)} \, dx \, d\eta}{\displaystyle\int_{-\infty}^{\infty} \int_{x_L(\eta)}^{x_H(\eta)} \underbrace{f(x)}_{N(0,1)} \underbrace{f(\eta \mid x)}_{N(\eta; x, \sigma_u^2)} \underbrace{f\big(z_b^*(\eta) \mid x\big)}_{N(z_b^*(\eta); x, \sigma_b^2)} \, dx \, d\eta}. \tag{27}$$

Example Equilibria. Figure 1 plots the equilibrium support rates of the dispersed shareholder (blue) and the blockholder (red) as the precision of the public signal varies with σ_u. Across all panels, the ownership structure $(\Psi_D, \psi_b) = (0.8, 0.2)$, the information parameters $(\sigma_D, \sigma_b) = (5.0, 5.0)$, and the passing threshold $\lambda^* = 0.5$ remain fixed; the only difference across examples is the preference pair (δ_D, δ_b).

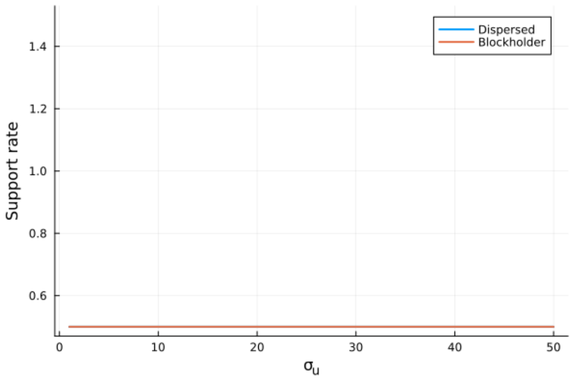
When both shareholders are unpreferenceed, $(\delta_D, \delta_b) = (0, 0)$, the two agents adopt identical cutoff rules. Their equilibrium support rates coincide and remain constant across all levels of σ_u, indicating that, absent preference heterogeneity, variation in public information precision alone does not generate differences in voting behavior.

Introducing a small preference for the blockholder, $(\delta_D, \delta_b) = (0, 0.1)$, creates only minor differences in observed support. At intermediate levels of public noise, the two curves nearly overlap because strategic interactions offset preference heterogeneity: even with a stronger pro-management inclination, the blockholder may appear no more supportive than the dispersed shareholder once pivotality and information precision are jointly accounted for.

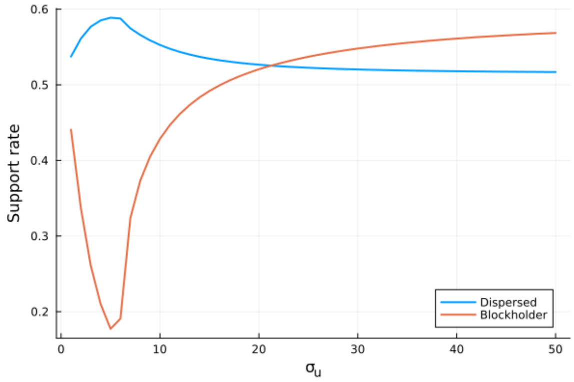
When the blockholder is slightly more pro-management $(\delta_D, \delta_b) = (0, 0.1)$, the two share-

Figure 1. Equilibrium support rates as a function of public-signal noise σ_u.
This figure displays equilibrium support rates of dispersed (blue) and blockholder (orange) shareholders, illustrating that differences in observed support rates cannot be directly interpreted as differences in preference preference, as strategic and signal effects jointly determine voting. In each subfigure, $(\Psi_D, \psi_b) = (0.8, 0.2)$ and $\lambda^* = 0.5$.

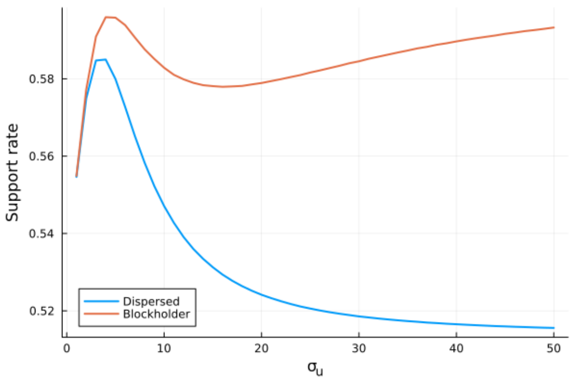
(a) No preference: $(\delta_D, \delta_b) = (0, 0)$. Support rates coincide and remain constant across σ_u.



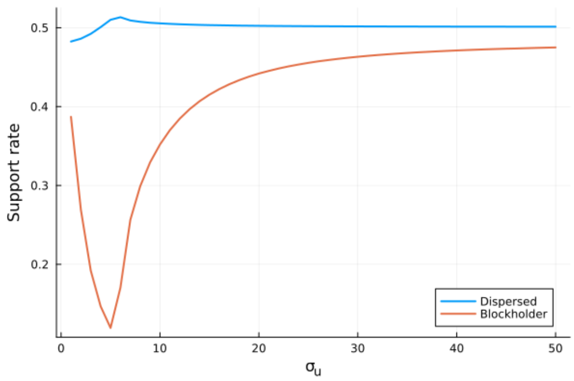
(b) Mild blockholder preference: $(\delta_D, \delta_b) = (0, 0.1)$. Non-monotonic strategic response to σ_u.



(c) Equal preferencees: $(\delta_D, \delta_b) = (0.1, 0.1)$. Blockholder support remains weakly higher.



(d) Dispersed less favorable: $(\delta_D, \delta_b) = (-0.1, 0)$. Blockholder support is consistently lower.



holders' support rates respond very differently to the precision of public information. For small values of σ_u (highly precise public signals), the dispersed shareholder's support rate exceeds that of the blockholder. In this region, the public signal dominates private information, and the blockholder's larger ownership share makes her vote pivotal only when the dispersed support rate is near the passing threshold—states typically associated with lower underlying proposal quality x_j. As the public signal becomes noisier, the blockholder's decision relies more on her private signal and preference. Her support rate initially falls, reaching a minimum where pivotality and information noise interact most strongly, before rising again as her private information becomes the dominant driver of beliefs.

In the limit of very imprecise public information (large σ_u), both shareholders' votes depend almost entirely on their own private signals. The blockholder's mild pro-management preference then leads to a slightly higher overall support rate. The resulting non-monotonic pattern of the blockholder's curve thus illustrates the trade-off between information precision and strategic pivotality: when the public signal is moderately noisy, pivotality suppresses her support, but as the public signal loses informational content, preference once again becomes the decisive force behind voting behavior.

When both shareholders are pro-management, $(\delta_D, \delta_b) = (0.1, 0.1)$, the blockholder's support rate is uniformly higher across the entire range of σ_u. In this case, differences in information precision and ownership concentration amplify rather than offset the preference, leading to consistently greater support from the blockholder.

Finally, when the dispersed shareholder is slightly anti-management, $(\delta_D, \delta_b) = (-0.1, 0)$, the pattern reverses: the blockholder's support rate remains below that of the dispersed shareholder for all σ_u. This occurs because the blockholder's pivotality interacts with the information environment. As public signals become noisier, she votes "For" only in states where the dispersed support rate is close to the passing threshold—states that tend to correspond to lower proposal quality—thus reducing her overall support.

Taken together, these patterns demonstrate that observed support rates do not map monotonically to shareholders' latent preferencees. Strategic and informational effects can either mask or exaggerate underlying preferences depending on the information structure. Therefore, it is generally invalid to infer an investor's preference directly from her average voting support, a central insight motivating the structural estimation that follows.

4. Estimation

A key distinction between the theoretical model and the empirical implementation arises from the econometrician's information set. In the model, all shareholders observe the true public signal η_j, which directly enters their posterior beliefs and voting decisions. Consequently, the analytical expressions for posterior densities in Section 2 condition explicitly on η_j, and there is no need to integrate over this variable. In contrast, the econometrician does not observe

η_j directly but only the binary ISS recommendation, $\text{ISS}_j = \mathbf{1}\{\eta_j > \xi\}$. This information loss implies that the true realization of η_j is latent from the econometrician's perspective and known only to belong to a range,

$$\eta_j \in \mathcal{I}(\text{ISS}_j) = \begin{cases} (-\infty,\ \xi], & \text{ISS}_j = 0, \\ \\ (\xi,\ \infty), & \text{ISS}_j = 1. \end{cases} \tag{28}$$

As a result, the econometrician must integrate over all possible realizations of the public signal consistent with the observed ISS outcome. Although there is no such integration in the agents' problem, this additional averaging step is crucial in estimation. The integral over η effectively replaces the conditioning on the exact signal in the model and weights each feasible η by its likelihood $f(\eta \mid x_j)$. Intuitively, this adjustment ensures that the estimation accounts for the fact that shareholders condition their votes on the true η_j, while the econometrician only observes the coarser partition induced by ISS. This is the key difference between the theoretical posterior densities in equations (8) and (10) and their econometric counterparts (8^{ISS}) and (10^{ISS}), which form the basis for the empirical recovery of shareholder biases.

4.1. First stage: Information structure and cutoffs

The first-stage estimation proceeds in two steps: first, using dispersed shareholders' voting records, we estimate the information and preference parameters for dispersed shareholders; second, we estimate the blockholders' information and cutoff parameters.

4.1.1. Dispersed shareholders

Mapping from the voting rule to observed outcomes. Under the dispersed-shareholder voting rule,

$$\tau_j = \Phi\big(A_D \eta_j + B_D x_j - C_D\big), \qquad A_D = \frac{\sigma_{\varepsilon D}}{\sigma_u^2}, \quad B_D = \frac{1}{\sigma_{\varepsilon D}}, \quad C_D = \sigma_{\varepsilon D}\, \Lambda_D k_D,$$

with $\Lambda_D = 1 + 1/\sigma_u^2 + 1/\sigma_{\varepsilon D}^2$. This relationship is nonlinear in τ_j but monotonic, so we apply the probit transform

$$q_j = \Phi^{-1}(\tau_j),$$

to linearize the relationship between the observed support rate and the underlying signals. Conditional on x_j and η_j, the transformed variable q_j is affine in the latent public signal plus an independent disturbance arising from private-signal noise:

$$\Phi^{-1}(\tau_j) = a\eta_j + b + \varepsilon_q, \qquad \varepsilon_q \sim N(0, v), \tag{29}$$

where

$$a = \frac{\sigma_{\varepsilon D}}{\sigma_u^2} + \frac{1}{\sigma_{\varepsilon D}(1+\sigma_u^2)}, \qquad b = -\,\sigma_{\varepsilon D}\, k_D \left(1 + \frac{1}{\sigma_u^2} + \frac{1}{\sigma_{\varepsilon D}^2}\right), \qquad v = \frac{\sigma_u^2}{(1+\sigma_u^2)\sigma_{\varepsilon D}^2}.$$

The public signal itself is normally distributed,

$$\eta_j \sim N(0, \sigma_\eta^2), \qquad \sigma_\eta^2 = 1 + \sigma_u^2,$$

reflecting both the fundamental uncertainty about proposal quality and the additional public-signal noise.

Truncated information from ISS recommendations. The econometrician does not observe η_j directly but only whether it exceeds the ISS threshold ξ, thus, the distribution of η_j is truncated from above or below depending on the observed ISS recommendation. Intuitively, $\text{ISS}_j = 1$ indicates that the public signal favored supporting the proposal, while $\text{ISS}_j = 0$ indicates the opposite. Because the observed τ_j reflects both the latent public signal and private information, estimation proceeds by integrating over the latent η_j within the region consistent with the observed ISS gate.

Likelihood function. Combining equations (29)–(28), the joint density of (q_j, η_j) is bivariate normal conditional on θ_D. The contribution of each proposal to the likelihood depends on whether η_j lies above or below ξ.

For proposals with $\text{ISS}_j = 1$ (upper tail, $\eta_j > \xi$), the joint density is

$$f(q_j, \text{ISS}_j = 1 \mid \theta_D) = \int_{\xi}^{\infty} \frac{1}{\sqrt{2\pi v}} \exp\left[-\frac{(q_j - a\eta - b)^2}{2v}\right] \cdot \frac{1}{\sigma_\eta} \phi\left(\frac{\eta}{\sigma_\eta}\right) \, d\eta, \tag{30}$$

where $\phi(\cdot)$ is the standard normal pdf. For proposals with $\text{ISS}_j = 0$ (lower tail, $\eta_j < \xi$), the joint density is

$$f(q_j, \text{ISS}_j = 0 \mid \theta_D) = \int_{-\infty}^{\xi} \frac{1}{\sqrt{2\pi v}} \exp\left[-\frac{(q_j - a\eta - b)^2}{2v}\right] \cdot \frac{1}{\sigma_\eta} \phi\left(\frac{\eta}{\sigma_\eta}\right) \, d\eta. \tag{31}$$

These two integrals correspond to the likelihood contributions conditional on the ISS gate—i.e., integrating over the unobserved range of public signals consistent with the observed ISS recommendation.

4.1.2. Blockholders

Overview and motivation. In the second stage, we estimate information and preference parameters for each blockholder b, conditional on the first-stage estimates of the public-signal parameters $(\hat{\sigma}_u, \hat{\xi})$. Blockholders differ from dispersed shareholders in two key ways. First, each blockholder b observes a possibly more precise private signal z_{bj} about proposal quality, with signal variance σ_b^2. Second, each blockholder's vote has a positive ownership weight $\psi_b > 0$, so its decision can be pivotal with positive probability. The goal of this stage is to recover, for each blockholder, the pair (σ_b, k_b) that best rationalizes their observed voting patterns across proposals.

Integrating out proposal quality. We do not observe proposal quality x_j directly. Conditional on the public signal, x_j is normally distributed as

$$x_j \mid \eta_j \sim N\left(\frac{\eta_j}{1 + \sigma_u^2}, \frac{\sigma_u^2}{1 + \sigma_u^2}\right).$$

Since $z_{bj} = x_j + \varepsilon_{bj}$ with $\varepsilon_{bj} \sim N(0, \sigma_b^2)$, integrating out x_j yields the blockholder's vote probability conditional on η_j:

$$\Pr(v_{bj} = 1 \mid \eta_j, \sigma_u, \sigma_b, k_b) = \Phi\left(\frac{C_1 \eta_j - \sigma_b k_b \Lambda_b}{D}\right), \tag{32}$$

where

$$C_1 \equiv \frac{\sigma_b}{\sigma_u^2} + \frac{1}{\sigma_b(1+\sigma_u^2)}, \qquad D \equiv \sqrt{1 + \frac{\sigma_u^2}{(1+\sigma_u^2)\sigma_b^2}}.$$

Equation (32) expresses the blockholder's probability of supporting the proposal as a probit function of the latent public signal.

Conditioning on the ISS recommendation. In the data, the public signal η_j is unobserved, but we know whether it exceeds the ISS threshold ξ. Since $\eta_j \sim N(0, \sigma_\eta^2)$ with $\sigma_\eta^2 = 1 + \sigma_u^2$, the econometrician faces two truncated-normal cases depending on ISS_j.

For proposals with $\text{ISS}_j = 1$ (upper tail), η_j is distributed as $\text{TruncNormal}(0, \sigma_\eta^2; [\xi, \infty))$, yielding

$$\Pr(v_{bj} = 1 \mid \text{ISS}_j = 1, \sigma_u, \sigma_b, k_b, \xi) = \frac{1}{1 - \Phi\left(\frac{\xi}{\sigma_\eta}\right)} \int_\xi^\infty \Phi\left(\frac{C_1 \eta - \sigma_b k_b \Lambda_b}{D}\right) \frac{1}{\sigma_\eta} \phi\left(\frac{\eta}{\sigma_\eta}\right) d\eta. \tag{33}$$

For proposals with $\text{ISS}_j = 0$ (lower tail), η_j follows $\text{TruncNormal}(0, \sigma_\eta^2; (-\infty, \xi])$:

$$\Pr(v_{bj} = 1 \mid \text{ISS}_j = 0, \sigma_u, \sigma_b, k_b, \xi) = \frac{1}{\Phi\left(\frac{\xi}{\sigma_\eta}\right)} \int_{-\infty}^\xi \Phi\left(\frac{C_1 \eta - \sigma_b k_b \Lambda_b}{D}\right) \frac{1}{\sigma_\eta} \phi\left(\frac{\eta}{\sigma_\eta}\right) d\eta. \tag{34}$$

These two integrals correspond to the probability that blockholder b votes in favor conditional on the ISS gate, integrating over the latent public-signal realizations consistent with the observed ISS recommendation.

Likelihood function for blockholder b. Let $\mathcal{J}_b$ denote the set of proposals on which blockholder b casts a vote, and let $v_{bj} \in \{0, 1\}$ be the observed vote. The model implies that, conditional on (σ_u, ξ), each vote is an independent Bernoulli draw with success probability given by (33) or (34), depending on the ISS status:

$$p_{bj}(\sigma_b, k_b) = \begin{cases} \Pr(v_{bj} = 1 \mid \text{ISS}_j = 1, \sigma_u, \sigma_b, k_b, \xi), & \text{if ISS}_j = 1, \\ \\ \Pr(v_{bj} = 1 \mid \text{ISS}_j = 0, \sigma_u, \sigma_b, k_b, \xi), & \text{if ISS}_j = 0. \end{cases}$$

The likelihood for blockholder b is therefore

$$L_b(\sigma_b, k_b \mid \sigma_u, \xi) = \prod_{j \in \mathcal{J}_b} [p_{bj}(\sigma_b, k_b)]^{v_{bj}} [1 - p_{bj}(\sigma_b, k_b)]^{1 - v_{bj}}.$$

4.2. Second stage: Back out biases

Transition and idea. With the first–stage estimates of the information primitives and public recommendation rule,

$$\widehat{\sigma}_u^2, \quad \widehat{\sigma}_{\varepsilon D}^2, \quad \{\widehat{\sigma}_{\varepsilon b}^2\}_{b=1}^{N_B}, \quad \widehat{\xi},$$

and the equilibrium cutoffs $\widehat{k}_D$ and $\{\widehat{k}_b\}_{b=1}^{N_B}$ in hand, we can recover shareholders' preference parameters (biases) δ_D and $\{\delta_b\}$. The key identification step is that, in equilibrium, each shareholder is indifferent at her cutoff *when pivotal.* In the data, however, we do not observe the public signal η_j directly. Instead we observe ISS's coarse recommendation $\text{ISS}_j = \mathbf{1}\{\eta_j > \xi\}$. Consequently, the pivotal posterior that appears in the indifference condition must integrate over all public signal realizations consistent with the observed ISS bucket. This is the only difference between the model expressions (which condition on η_j) and the estimating equations (which condition on ISS_j).

Dispersed shareholders. For a dispersed shareholder, the indifference condition reads

$$\mathrm{E}\big[x_j \,\big|\, z_{Dj} = z_D^\star(\eta),\ \text{ISS}_j,\ \text{PIV}_{Dj}\big] \;=\; -\,\delta_D, \quad z_D^\star(\eta) = \sigma_{\varepsilon D}^2 \left(\Lambda_D\, k_D - \frac{\eta}{\sigma_u^2} \right), \qquad \Lambda_D = 1 + \frac{1}{\sigma_u^2} + \frac{1}{\sigma_{\varepsilon D}^2}.$$

Because we only observe $\text{ISS}_j \in \{0, 1\}$, we average over the set of η consistent with ISS, weighting by the conditional density $f(\eta \mid x_j)$ implied by $\widehat{\sigma}_u^2$. Formally, the posterior density of x_j

under dispersed pivotality is

$$\lim_{N_D\to\infty} f\big(x_j \,\big|\, z_{Dj},\ \mathrm{ISS}_j,\ \mathrm{PIV}_{Dj}\big)$$
$$\propto\ f(x_j)\sum_{V_{Bj}}\int_{\eta\in\mathcal{I}(\mathrm{ISS}_j)} \underbrace{f(\eta\mid x_j)}_{\text{public-signal density}}\ \underbrace{f\big(z_{Dj}\mid x_j\big)}_{\text{signal effect}}\ \underbrace{\delta\big(p_D(x_j,\eta)-\tau^{V_{Bj}}\big)}_{\text{pivotal constraint}}\ \underbrace{\Pr\big(V_{Bj}\mid x_j,\eta\big)}_{\text{strategic effect}}\ d\eta,$$

where

$$p_D(x_j,\eta)=\Phi\left(\sigma_{\varepsilon D}\left[\frac{\eta}{\sigma_u^2}+\frac{x_j}{\sigma_{\varepsilon D}^2}-k_D\Lambda_D\right]\right),\qquad \tau^{V_{Bj}}=\frac{\lambda^*-\Psi_{\mathrm{yes}}(V_{Bj})}{\Psi_D}.$$

The Dirac delta enforces the dispersed knife–edge condition $p_D(x_j,\eta)=\tau^{V_{Bj}}$. Collapsing along this knife–edge yields the evaluated form used in computation:

$$f\big(x_j \,\big|\, z_{Dj},\ \mathrm{ISS}_j,\ \mathrm{PIV}_{Dj}\big)$$
$$\propto\ \sum_{V_{Bj}}\int_{\eta\in\mathcal{I}(\mathrm{ISS}_j)} \underbrace{f(\eta\mid x_j)}_{\text{public-signal weight}}\ \underbrace{f\big(x_\eta^{V_{Bj}}\big)}_{\text{prior}}\ \underbrace{f\big(z_{Dj}\mid x_\eta^{V_{Bj}}\big)}_{\text{signal effect}}\ \underbrace{\chi^{V_{Bj}}}_{\text{Jacobian}}\ \underbrace{\Pr\big(V_{Bj}\mid x_\eta^{V_{Bj}},\eta\big)}_{\text{strategic effect}}\ d\eta,$$

with

$$x_\eta^{V_{Bj}}=\sigma_{\varepsilon D}^2\left(\Lambda_D k_D-\frac{\eta}{\sigma_u^2}+\frac{q^{V_{Bj}}}{\sigma_{\varepsilon D}}\right),\qquad \chi^{V_{Bj}}=\frac{\sigma_{\varepsilon D}}{\phi(q^{V_{Bj}})},\qquad q^{V_{Bj}}=\Phi^{-1}(\tau^{V_{Bj}}).$$

Plugging into the indifference condition delivers the sample analogue for δ_D:

$$\widehat{\delta}_D\ =\ -\mathrm{E}_{\widehat{\theta}}\big[x_j \,\big|\, z_{Dj}=z_D^\star(\eta),\ \mathrm{ISS}_j,\ \mathrm{PIV}_{Dj}\big],$$

where expectations are taken under the first–stage parameter vector $\widehat{\theta}=(\widehat{\sigma}_u^2,\widehat{\sigma}_{\varepsilon D}^2,\{\widehat{\sigma}_{\varepsilon b}^2\},\widehat{\xi},\widehat{k}_D,\{\widehat{k}_b\})$.

Blockholders. For blockholder b, pivotality occurs when the remaining shares put the outcome in a ψ_b–wide band. Let $\Psi_{\mathrm{yes}}(V_{B,-b,j})$ be the yes–weight of other blockholders and

$$\tau_L^{V_{B,-b,j}}=\frac{\lambda^*-\Psi_{\mathrm{yes}}(V_{B,-b,j})-\psi_b}{\Psi_D},\qquad \tau_H^{V_{B,-b,j}}=\frac{\lambda^*-\Psi_{\mathrm{yes}}(V_{B,-b,j})}{\Psi_D}.$$

The blockholder's pivotal posterior (conditioning on the ISS bucket) is

$$\lim_{N_D\to\infty} f\big(x_j \,\big|\, z_{bj},\ \mathrm{ISS}_j,\ \mathrm{PIV}_{bj}\big)$$
$$\propto\ f(x_j)\ \underbrace{f\big(z_{bj} \mid x_j\big)}_{\text{signal effect}} \sum_{V_{B,-b,j}} \int_{\eta\in\mathcal{I}(\mathrm{ISS}_j)} \underbrace{f(\eta \mid x_j)}_{\text{public-signal density}}\ \underbrace{\mathbf{1}\Big\{\ p_D(x_j,\eta)\in[\tau_L^{V_{B,-b,j}},\tau_H^{V_{B,-b,j}}]\ \Big\}}_{\text{pivotal window in } x}\ \underbrace{\Pr\big(V_{B,-b,j} \mid x_j,\eta\big)}_{\text{strategic effect}}\ d\eta.$$

Equivalently, solve $p_D(x_j,\eta) = \tau_{\boldsymbol{\cdot}}^{V_{B,-b,j}}$ for x to obtain the implied x–window

$$x_{L,\eta}^{V_{B,-b,j}} = \sigma_{\varepsilon D}^2\left(\Lambda_D k_D - \frac{\eta}{\sigma_u^2} + \frac{q_L^{V_{B,-b,j}}}{\sigma_{\varepsilon D}}\right), \quad x_{H,\eta}^{V_{B,-b,j}} = \sigma_{\varepsilon D}^2\left(\Lambda_D k_D - \frac{\eta}{\sigma_u^2} + \frac{q_H^{V_{B,-b,j}}}{\sigma_{\varepsilon D}}\right),$$

so the pivotal restriction is the indicator $\mathbf{1}\{x_j \in [x_{L,\eta}^{V_{B,-b,j}}, x_{H,\eta}^{V_{B,-b,j}}]\}$. The blockholder's indifference condition becomes

$$\mathrm{E}\big[x_j \,\big|\, z_{bj} = z_b^\star(\eta),\ \mathrm{ISS}_j,\ \mathrm{PIV}_{bj}\big] \ =\ -\,\delta_b, \qquad z_b^\star(\eta) = \sigma_{\varepsilon b}^2\left(\Lambda_b\, k_b - \frac{\eta}{\sigma_u^2}\right), \quad \Lambda_b = 1 + \frac{1}{\sigma_u^2} + \frac{1}{\sigma_{\varepsilon b}^2},$$

which we evaluate by integrating x over the pivotal window, averaging η over $\mathcal{I}(\mathrm{ISS}_j)$, and weighting by the profile likelihood over the other blockholders' votes (all at first–stage parameters). The estimator is

$$\widehat{\delta}_b \ =\ -\,\mathrm{E}_{\widehat{\theta}}\big[x_j \,\big|\, z_{bj} = z_b^\star(\eta),\ \mathrm{ISS}_j,\ \mathrm{PIV}_{bj}\big], \qquad b = 1,\ldots,N_B.$$

Summary of Estimation. Taken together, the two stages of estimation recover all structural primitives of the model: the information parameters governing public and private signal precisions $(\sigma_u, \sigma_{\varepsilon i})$, the voting cutoffs (k_i) that rationalize observed support rates, and the preference parameters (δ_i) that capture each shareholder's stance toward management proposals. The first stage identifies the information structure by matching the cross-sectional variation in observed and simulated support rates of dispersed and block shareholders conditional on ISS recommendations, while the second stage inverts the pivotality conditions to recover the bias parameters that rationalize those cutoffs. Importantly, the second stage explicitly integrates over the latent public signal η_j conditional on the econometrician's observed ISS recommendation, thereby translating the model's information environment into an estimable form. This procedure yields shareholder-specific measures of informational precision and preference heterogeneity that are consistent with equilibrium strategic voting under incomplete information. The recovered parameters serve as the foundation for the subsequent counterfactual analyses.

5. Results

5.1. Institutional Background and Hypothesis Development

We use the model to estimate the voting preferences of the largest institutional investors in the US. We focus particularly on the Big Three, due to the size and breadth of their portfolios. For example, Vanguard appears among the largest five shareholders in every S&P 500 firm, where its average stake is ten percent.

As blockholders, the Big Three have an outsized influence on shareholder voting. In general, each asset manager maintains the voting rights of the shares they manage and votes them using a centralized voting policy (Fichtner et al., 2017). Further, because retail investors' turnout rate is low Brav et al. (2022), the Big Three's effective voting power is often far larger than their nominal ownership stake. As such, in the typical proxy vote, more than twenty percent of a firm's shares are voted by just three institutions.

Empirically, passive funds vote according to management's recommendations more often than other types of shareholders, leading some to conclude that they are reluctant to confront firm management. For inpreference, Heath et al. (2022) show that passive funds are ten percent more likely to support management in close governance votes, arguing they "cede power to firm management" and "[worsen] the alignment between managers and shareholders." Furthermore, Brav et al. (2024) find a similar gap using proxy contests, suggesting that passive funds have an inherent preference to keep existing management teams in place rather than replace them with activist investors.[1]

Bebchuk and Hirst (2019) argue that these empirical patterns are consistent with the Big Three having incentives to be "excessively deferential" to firm management. For example,

[1]The authors stress, however, that passive funds do not appear to be unengaged shareholders, arguing that they express dissent through mechanisms other than supporting dissident shareholders outright.

business ties with their portfolio firms may lead the Big Three to adopt a pro-management voting policy (Davis and Kim, 2007; Cvijanović et al., 2016). In addition, the Big Three may be wary of political backlash spearheaded by corporate managers if they use their voting power aggressively (Roe, 1991). Beyond conflicts of interest, some have argued that large asset managers — including the Big Three — may have anticompetitive incentives if they own multiple firms in the same industry (Azar et al., 2018). From this perspective, the Big Three may be prone to support proposals that lead managers to compete less aggressively, such as compensation packages with low sensitivity to firm performance (Antón et al., 2023). Finally, the Big Three may simply have a distinct philosophy about corporate governance (Bubb and Catan, 2022; Bolton et al., 2020). Ultimately, we am agnostic about the exact reason why the Big Three's voting preferences might be different than other shareholders. our contribution is to accurately test for these differences in a manner that accounts for strategic effects.

our model suggests caution when using shareholders' voting behaviour as a measure of their underlying preferences. As the numerical examples from Section 3 show, there is an ambiguous relationship between the amount of proposals a blockholder believes should pass and the amount it supports in equilibrium. Because passive funds are managed by blockholders, it is not clear that their tendency to support management reveals that they are preferenceed. Therefore, we structurally estimate our model to test this hypothesis. In our model, a shareholder's preferences are summarized by its preference measure, so the Big Three are preferenceed when their preference measures are higher than the dispersed shareholders' preference measure. Thus, the main parameter of interest is the difference $\Phi(\delta_b) - \Phi(\delta_D)$.[2]

Some commentators have also suggested that the Big Three are uninformed voters. For example, Lund (2017) argues that the Big Three do not acquire firm-specific information when they trade because their goal is to replicate an index, rather than identify undervalued firms. Further, their low fees make any investment in acquiring information unprofitable. In our model, the degree to which a shareholder is informed is captured by the precision of its signal. Therefore, we also test whether the precision of the Big Three's signals and the dispersed shareholders' signals are equal, where the parameter of interest is the difference $\sigma_b^2 - (\sigma_\eta^2 + \sigma_D^2)$.

[2]We cannot test which shareholder has the correct preferences. We can identify relative preferences only, so our model tests whether the Big Three are preferenceed relative to the dispersed shareholders.

To estimate our model, we use a sample of compensation proposals, commonly known as "Say-on-Pay" votes, where shareholders are asked to approve the firm's executive compensation. Because these are management proposals, a "For" vote is an expression of support for management, while an "Against" vote is an expression of dissent. Although Say-on-Pay votes are non-binding, they are economically important. Institutional Shareholder Services, a leading proxy advisor, deems proposals receiving less than 70% support to warrant a response from management and considers recommending Against a director's reelection if the response is insufficient (Institutional Shareholder Services, 2021). Consistent with this guidance, Ertimur et al. (2013) find that low voting support is associated with firms changing their compensation practices.

5.2. Sample Construction

5.2.1. Data collection

We use the ISS Voting Results dataset to gather a universe of Say-on-Pay proposals held at annual meetings in 2020. We identify Say-on-Pay votes using the ISS Agenda Item ID "M0550." For each proposal, we collect identifying information about the firm, the date of the meeting, the number of shares voted for/Against/abstain, ISS's voting recommendation, and additional variables related to voting procedures. We keep only proposals with voting bases "For + Against + Abstain" or "For + Against." We merge share price information from the CRSP monthly stock file, matching on the month-end prior to the meeting date. We compute the proposal's total support rate, λ_j, following:

$$\lambda_j = \begin{cases} \dfrac{\text{\# shares for}}{\text{\# shares for} + \text{\# shares Against}}, & \text{if base = "For + Against,"} \\[2ex] \dfrac{\text{\# shares for}}{\text{\# shares for} + \text{\# shares Against} + \text{\# shares abstain}}, & \text{if base = "For + Against + Abstain".} \end{cases} \tag{38}$$

To focus on contentious votes, we follow the approach in Bubb and Catan (2022) and Bolton et al. (2020) and filter to proposals receiving less than 95% support. Next, we compute turnout,

χ_j, following:

$$\chi_j = \begin{cases} \dfrac{\text{\# shares for + \# shares Against}}{\text{\# shares outstanding}}, & \text{if base = ``For + Against,''} \\ \dfrac{\text{\# shares for + \# shares Against + \# shares abstain}}{\text{\# shares outstanding}}, & \text{if base = ``For + Against + Abstain''.} \end{cases} \tag{39}$$

We drop proposals with less than 50% turnout. For each Say-on-Pay proposal found in the ISS Voting Analytics dataset, we download all mutual fund voting records. We collect the name and ISS identifier of the mutual fund, the mutual fund family, and the vote cast. We keep only for/Against/abstain votes. We aggregate fund-level votes to the family-level, deeming a family to vote for the proposal if a majority of its funds support it. We download institutional ownership data from the Thomson Reuters 13F dataset, merging on the quarter prior to the meeting date. We compute each manager's ownership by dividing the number of shares held by the number of shares outstanding.

5.2.2. Estimation dataset

We now turn to how we map the raw data to the structure assumed in the model. First, we compute each 13F manager's turnout-adjusted ownership by dividing their raw ownership by the turnout rate χ_j. We deem a manager to be a blockholder if its turnout-adjusted ownership is at least five percent. Next, we match 13F manager names to ISS fund family names to record each blockholder's vote.[3] For each proposal, this step produces a set of blockholders, their votes, and their turnout-adjusted ownership. We construct the vector of blockholder ownership, ψ^{χ}_{Bj}, and the vector of blockholder votes, V^B_j, where the number of blockholders N_B is the count of unique blockholders across all proposals. Given the total support rate, λ_j, we impute the dispersed shareholders' support rate, τ_j, following

$$\tau_j = \frac{\lambda_j - V_j^{B\top} \psi^{\chi}_{Bj}}{\Psi^{\chi}_{Dj}}. \tag{40}$$

[3]In the underlying data, we keep only for/Against/abstain votes; fund-level votes are aggregated to the family level.

Table 2. Summary Statistics

This table summarizes the sample of Say-on-Pay proposals we use to estimate the model. In Panel A, we provide statistics at the proposal-level. Turnout is the number of shares voted in the proposal divided by the number of shares outstanding. Number of blockholders is the count of blockholders that own the firm and whom we can match a vote to. we define a blockholder as a shareholder who owns more than five percent of the firm's shares on a turnout-adjusted basis. To compute turnout-adjusted ownership, we divide raw ownership by the turnout rate. Blockholder ownership is the sum of each blockholder's raw ownership. Turnout-adjusted blockholder ownership is the sum of each blockholder's turnout- adjusted ownership. Support rate: total is the number of shares voted in support of the proposal divided by the number of votes cast. Support rate: dispersed shareholders is the percent of dispersed shareholders supporting the proposal, imputed using the method described in the text. Say-on-Pay passes is a dummy variable equal to one if the proposal's support rate exceeds the 70% passing threshold. Vote result is close is a dummy variable equal to one if the proposal's support rate is between 60% and 80%. In Panel B, we provide statistics at the blockholder-level. Mean ownership is the blockholder's average ownership across all proposals. Support rate is the percent of proposals supported by the blockholder. Pivot probability is the percent of proposals where had the blockholder changed its vote, the proposal would have passed (failed) if it actually failed (passed).

Panel A: proposal-level	Mean	Median	SD
Number of proposals		993	
Turnout	77.25%	78.87%	11.22%
Number of blockholders	2.95	3	1.14
Blockholder ownership	22.93%	22.80%	10.43%
Turnout-adj. blockholder ownership	29.53%	28.92%	12.86%
Support rate: total	83.65%	89.53%	14.11%
Support rate: dispersed shareholders	81.56%	87.12%	14.40%
Say-on-Pay passes	85.90%		
Vote result is close	18.03%		
Panel B: blockholder-level	Vanguard	BlackRock	State Street
Number of votes	909	800	291
Average ownership	8.25%	9.02%	4.98%
Average turnout-adjusted ownership	10.65%	11.62%	6.63%
Support rate	90.21%	95.00%	90.38%
Pivot probability	12.54%	17.00%	4.12%

Here, Ψ^{χ}_{Dj} denotes the dispersed shareholders' turnout-adjusted ownership share in proposal j. This step produces a series of voting results $\{\tau_j, V^B_j\}$ and ownership structures $\{\Psi_j\}$ suitable as inputs to the procedure described in Section IV.

5.2.3. Sample summary

We summarize our sample of 993 Say-on-Pay votes in Table 2. The average firm is owned by three blockholders with combined ownership of 23%, or 30% once adjusting for turnout; average turnout is 77%. On average, 84% of votes are cast in favour of the proposal. This decreases to 82% when restricting to votes cast by the dispersed shareholders. Under the

Table 3. Estimated Information Structure and Preference Parameters

This table reports the estimated parameters of the information structure and voting behavior. σ_u and ξ describe the public signal observed by all shareholders and the ISS recommendation threshold, respectively. For the dispersed shareholders and each blockholder, σ_i denotes the standard deviation of the private signal noise, k_i is the voting cutoff that captures strategic behavior, and δ_i represents the shareholder's preference or preference toward management. Standard errors are reported in parentheses below each point estimate.

			Blockholders		
	ISS	Dispersed	Vanguard	Blackrock	State Street
Public signal noise σ_u	9.900				
	(1.391)				
ISS cutoff ξ	10.283				
	(2.242)				
Private signal noise $\sigma_{\varepsilon i}$		4.918	14.490	11.925	11.901
		(1.228)	(4.169)	(4.445)	(4.120)
Cutoff k_i		-0.093	-0.028	-0.100	-0.013
		(0.013)	(0.013)	(0.021)	(0.028)
Preference δ_i		0.094	-0.152	-0.266	0.116
		(0.054)	(0.084)	(0.157)	(0.054)

convention that less than 70% support is considered a failure, 14% of proposals fail. This is the benchmark used by ISS to measure shareholder dissatisfaction, and the accepted benchmark for a passing SOP vote in the data, so we set the passing threshold, λ^*, to 70% when we estimate the model (Institutional Shareholder Services, 2021).

We summarize the Big Three's ownership and voting behavior in Panel B. Vanguard is the most frequent blockholder, appearing in 91% of all proposals, while BlackRock appears in 80%. State Street appears as a blockholder in one third of all proposals; it meets the five percent threshold less often because it is smaller. Vanguard and BlackRock own $\approx$ 9% of the average firm, which increases to $\approx$ 12% once adjusting for turnout. Relative to the dispersed shareholders, each of the Big Three is more likely to support management. BlackRock supports 95% of all proposals, higher than the dispersed shareholders' average support rate. Vanguard and State Street are slightly less supportive at 90%, but still more likely to support management. These support rates are consistent with previous studies (e.g. Heath et al., 2022).

5.3. Estimation Results

Table 3 reports the estimated parameters of the information structure and individual cutoffs. Together, these estimates provide a quantitative mapping from the information environment to observed voting behavior, allowing us to assess whether the model's central strategic mechanism is consistent with the data.

The estimated public signal variance σ_u^2 is large (with $\sigma_u \approx 9.9$), implying that the ISS recommendation—driven by the public signal threshold $\xi = 10.3$—is a relatively coarse but influential source of information. In contrast, dispersed shareholders possess moderately precise private signals ($\sigma_{\varepsilon D} \approx 4.9$), while blockholders' private signals are much noisier, ranging from $\sigma_{\varepsilon b} \approx 3.8$ for the most informed (State Street) to over 21 for the least informed (BlackRock). This structure mirrors the institutional environment: large passive funds rely heavily on public information and ISS guidance, whereas the dispersed mass of smaller investors effectively aggregates more idiosyncratic private signals.

Table 4. Voting frequencies by shareholder and ISS recommendation

Shareholder	All	ISS=0	ISS=1
Dispersed	0.816	0.618	0.861
Vanguard	0.902	0.518	0.991
BlackRock	0.950	0.734	0.997
State Street	0.904	0.457	0.965

It might be surprising that private-signal variances are larger for blockholders than for dispersed shareholders, but this pattern is consistent with the observed voting frequencies. Table 4 reports the conditional support rates given ISS recommendations. In the data, blockholders' support rates respond much more sharply to the ISS recommendation than do those of dispersed shareholders: for instance, Vanguard's support rate rises from 0.52 when ISS recommends "Against" to 0.99 when ISS recommends "For," whereas the corresponding increase for dispersed shareholders is from 0.62 to 0.86. A larger variance of the private signal implies that blockholders' votes depend more heavily on the public signal, making their support rates more sensitive to changes in ISS recommendations. Thus, while the higher estimated noise among blockholders may appear surprising given their sophistication, it is consistent with the empirical pattern of stronger conditional variation in their observed voting behavior.

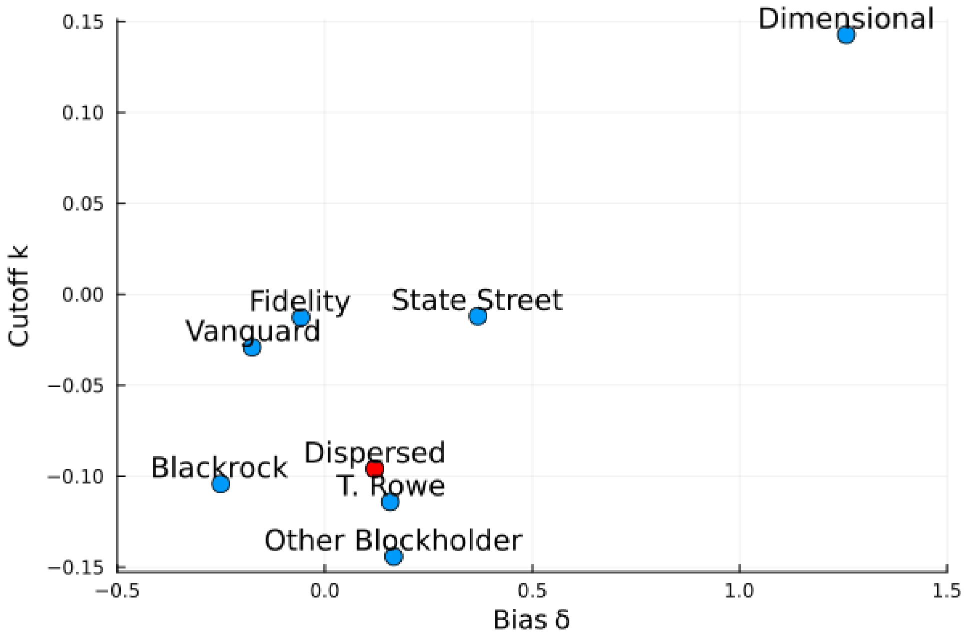
Cutoff estimates k_i and preference parameters δ_i jointly determine the strategic posture of each shareholder type. For dispersed shareholders, the cutoff $k_D = -0.093$ and preference $\delta_D = 0.094$ together imply a modestly favorable preference toward management, consistent with their relatively high observed support rates (0.86 when ISS recommends "For," 0.62 when "Against"). State Street, with the smallest private noise ($\sigma_{\varepsilon b} = 11.9$) and a mild pro-management preference ($\delta_b = 0.116$), shows a similar pattern of high conditional support but greater sensitivity to the public signal—nearly unanimous support when ISS recommends "For" and a sharp decline when ISS recommends "Against."

By contrast, Vanguard and BlackRock display markedly higher cutoffs and negative preference parameters ($\delta_b = -0.15$ and -0.27, respectively), suggesting a stricter voting standard. Despite these more skeptical priors, their unconditional support rates remain high when ISS recommends approval (0.99 and 0.997), but fall sharply when ISS is unfavorable (0.52 and 0.73). This asymmetry reflects the key strategic channel of the model: when the ISS signal is negative, blockholders' priors shift downward, and the high cutoffs imply that they will support management only in states where private information is strongly positive. In expectation, such states are rare, leading to a low conditional support rate even for shareholders with substantial voting power. Conversely, when ISS is favorable, the same strict cutoff translates into near-universal support because the posterior mean shifts upward for all investors.

Vanguard's particularly high estimated signal noise ($\sigma_{\varepsilon b} = 14.5$) implies heavy reliance on the public signal, consistent with its near-perfect correlation with ISS recommendations. BlackRock, though somewhat better informed ($\sigma_{\varepsilon b} = 11.9$), shows a similarly polarized pattern, but with a slightly higher support rate under ISS opposition. Both cases illustrate that a shareholder's information precision and preference interact nonlinearly: weak private information amplifies the public-signal effect, while high cutoffs magnify the asymmetry between "For" and "Against" recommendations.

Overall, these results are consistent with the model's claim that observed support rates cannot be interpreted as direct evidence of underlying preference. State Street, with a small

Figure 2. Estimated Cutoffs and Biases
The figure plots each shareholder's estimated cutoff k_i Against preference δ_i. Cutoffs and preferencees are positively correlated, consistent with the pivotality condition $E[x_j \mid \eta_j, m_{ij} = k_i, \text{piv}_{ij}] = -\delta_i$. Because higher cutoffs imply lower support rates, the figure illustrates that more pro-management shareholders may appear less supportive in equilibrium.



preference and precise private signal, exhibits moderate support because strategic inference leads it to vote "Against" in marginal proposals where it is likely pivotal. Vanguard and BlackRock, despite having more negative preference parameters, appear more supportive in equilibrium precisely because their information is noisier and their votes move in tandem with the public signal. Thus, the mapping from estimated preferences (δ_i) to observed support is not monotonic: an investor with a stronger pro-management preference can appear less supportive in the data once the informational and strategic structure of voting is taken into account.

Figure 2 plots the estimated individual cutoffs k_i Against the inferred preference parameters δ_i for both dispersed and block shareholders. The estimates reveal a clear positive relationship between the two: investors with higher inferred preferencees toward management tend to adopt higher cutoffs. This pattern is consistent with the model's equilibrium condition $E[x_j \mid \eta_j, m_{ij} = k_i, \text{piv}_{ij}] = -\delta_i$, which implies that more pro-management shareholders (higher δ_i) are willing to approve proposals under states of lower expected quality, leading to higher k_i in equilibrium. The figure therefore visually confirms the central comparative static embedded in the model—preference and cutoff move in the same direction.

However, since the equilibrium support rate decreases in the cutoff k_i, the positive relation between δ_i and k_i implies a negative relation between the observed support rate and the underlying preference. In other words, more pro-management shareholders can appear *less supportive* in the data once strategic inference is taken into account. This is precisely the mechanism highlighted by the model: being pivotal provides information about the underlying proposal quality, and more preferenceed shareholders are pivotal only in relatively worse states. While the overall pattern is monotonic, the bottom-left corner of the figure—where less preferenceed but highly noisy investors such as BlackRock and Vanguard cluster—shows mild deviations, illustrating that information precision also shapes the mapping between preference and observed support. Together, the estimates demonstrate that equilibrium support reflects a joint outcome of preference, information, and strategic inference, rather than a simple one-to-one correspondence between pro-management sentiment and approval behavior.

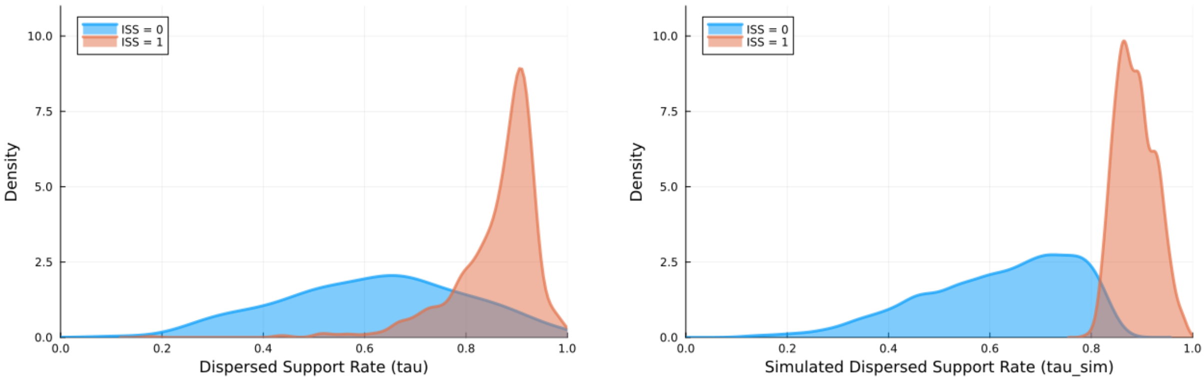
5.4. Model fit

Model Fit for Dispersed Support Rates. Figure 3 compares the empirical and model-implied distributions of the dispersed shareholders' support rate τ conditional on ISS recommendations. We show that the model replicates the conditional distributions in the data very closely. When ISS recommends "For" (ISS = 1), both the empirical and simulated distributions are sharply concentrated near one, reflecting strong dispersed support for management proposals. When ISS recommends "Against" (ISS = 0), both distributions shift leftward and exhibit substantially greater dispersion, capturing the broader range of shareholder disagreement observed in the data. The model accurately matches the conditional moments of the data: the mean and variance of the support rate are exactly the same as in the data. Overall, the model successfully reproduces the level, dispersion, and conditional shifts of dispersed shareholders' support rates, indicating an excellent fit to the empirical distributions.

Model Fit for Blockholder Support Rates. Table 5 compares the empirical and model-implied support rates of the three major blockholders conditional on ISS recommendations. For each blockholder, the model matches the observed conditional support rates almost perfectly. This close correspondence arises by construction: each blockholder's parameters (k_b, σ_b) are identified using two conditional moments—namely, the mean support rate conditional on

Figure 3. Model Fit of Conditional Dispersed Support Rates by ISS Recommendation
This figure compares the empirical (left panel) and model-implied (right panel) distributions of the dispersed shareholders' support rate τ conditional on ISS recommendations. The model reproduces the sharp rightward concentration of support rates when ISS recommends "For" (ISS = 1) and the broader, left-shifted distribution when ISS recommends "Against" (ISS = 0).

(a) Data **(b)** Simulation



ISS = 1 and on ISS = 0. The excellent fit in Table 5 therefore confirms that the estimated (k_b, σ_b) parameters successfully replicate the observed voting behavior of each blockholder under different ISS recommendations. These conditional fits provide a strong foundation for the second-stage estimation of blockholders' underlying preferences, which builds on these information structure parameters to separate informational and preference-driven components of their observed support.

5.5. Decomposition

To disentangle the roles of preferences, information and strategic behavior in shaping voting outcomes, we decompose each shareholder's support rate into four counterfactual environments. First, we compute the support rate when the only information is the prior $x_j \sim N(0, 1)$ (no private or common signals) and shareholders vote solely according to preference. Second, we introduce only the public signal while holding behavior non-strategic to isolate the influence of common information. Third, we allow both public and private signals but still assume non-strategic voting to capture the effect of private information. Finally, we incorporate strategic inference, where shareholders anticipate pivotality and adjust their cutoff rules accordingly.

Table 6 reports the predicted support rates of dispersed and block shareholders under

Table 5. Model Fit of Blockholder Support Rates Conditional on ISS Recommendation
This table compares the empirical and model-implied support rates of the three major blockholders conditional on ISS recommendations.

Blockholder ID	ISS {0,1}	Actual Rate	Simulated Rate
Vanguard	0	0.5176	0.5184
	1	0.9905	0.9905
BlackRock	0	0.7343	0.7366
	1	0.9970	0.9970
State Street	0	0.4571	0.4597
	1	0.9648	0.9648

the four counterfactual environments described above. Each case corresponds to a distinct information structure and behavioral rule. In Case 1, shareholders receive no information and vote "For" whenever the latent utility difference is positive,

$$V_{ij} = 1 \quad \Longleftrightarrow \quad x_j + \delta_i > 0,$$

which implies an unconditional support rate of $\Pr(V_{ij} = 1) = \Phi(\delta_i)$ given $x_j \sim \mathcal{N}(0, 1)$. Case 2 introduces a public signal $\eta_j = x_j + u_j$, so that voting depends on the posterior mean $E[x_j \mid \eta_j]$ but not on pivotality,

$$V_{ij} = 1 \quad \Longleftrightarrow \quad E[x_j \mid \eta_j] + \delta_i > 0.$$

Case 3 adds an idiosyncratic private signal $z_{ij} = x_j + \varepsilon_{ij}$, leading each shareholder to vote based on the posterior mean

$$m_{ij} = E[x_j \mid \eta_j, z_{ij}] = \frac{\dfrac{\eta_j}{\sigma_u^2} + \dfrac{z_{ij}}{\sigma_{\varepsilon_i}^2}}{1 + \dfrac{1}{\sigma_u^2} + \dfrac{1}{\sigma_{\varepsilon_i}^2}},$$

and to support the proposal if $m_{ij} > -\delta_i$. Finally, Case 4 embeds full strategic behavior: shareholders anticipate the possibility of being pivotal and endogenously adjust their cutoff k_i according to

$$E[x_j \mid \eta_j,\, m_{ij} = k_i,\, \mathrm{piv}_{ij}] = -\delta_i,$$

so that the equilibrium support rate is $\Pr(m_{ij} > k_i)$ given the estimated $(\sigma_u, \sigma_{\varepsilon_i}, \xi)$.

The quantitative results reveal how information precision and strategic inference jointly shape voting patterns. For the dispersed shareholders, support is high in all cases, reflecting their smaller information advantage and limited influence on the voting outcome. When only the public signal is available (Case 2), dispersed support reaches 0.875, and becomes perfectly aligned with the ISS recommendation: support is nearly one when ISS recommends approval and around 0.85 when it recommends rejection. Introducing private information (Case 3) slightly reduces overall support, as private noise introduces heterogeneity in posterior beliefs. Once strategic considerations are incorporated (Case 4), the dispersed shareholders' support falls modestly to 0.66 overall. The reason is that the pivotality correction effectively shifts their marginal cutoff upward—when a dispersed voter is pivotal, she infers that the proposal is likely in a borderline state, reducing the posterior expected quality of the proposal conditional on being decisive.

Among the blockholders, the patterns are more varied and economically revealing. For Vanguard and BlackRock, unconditional support rates are extremely low in the nonstrategic cases (0.048 and 0.009, respectively), despite their modest estimated preferencees. These low rates arise because, without strategic inference, both investors are swayed by the high noise in the public signal (σ_u) and their own relatively imprecise private information. Once strategic reasoning is introduced (Case 4), their support rises sharply—to 0.59 for Vanguard and 0.78 for BlackRock—and approaches unity when the ISS recommendation is favorable. This shift illustrates that conditioning on being pivotal provides additional information about the likely state of the world: if a blockholder finds herself pivotal, it must be that dispersed support lies in a narrow intermediate range, which tends to occur in moderately favorable proposal states. Hence, even though their underlying preference δ_i is not especially high, the strategic inference amplifies apparent support when the public signal is positive.

State Street, in contrast, displays the opposite adjustment. It appears almost fully supportive in the nonstrategic cases (support near one in Cases 2 and 3), but its equilibrium support rate falls to about 0.54 once strategic inference is incorporated. This decline arises because its more precise private signal (σ_{ε_b} relatively small) and moderate preference δ_b make the pivotal inference more adverse: when a well-informed shareholder like State Street is pivotal, it

Table 6. Decomposition of Support Rates Across Model Variants
This table reports predicted support rates under four informational and strategic environments: (1) no signals and no strategic behavior, (2) only public signal without strategic behavior, (3) both public and private signals without strategic behavior, and (4) both signals with strategic voting. Comparing across cases isolates the incremental effects of information and strategic inference on shareholders' approval behavior.

Shareholder	Condition	Case 1	Case 2	Case 3	Case 4
Dispersed	All	0.548	0.875	0.698	0.660
	ISS=1		1.000	0.908	0.887
	ISS=0		0.852	0.660	0.619
Vanguard	All	0.430	0.048	0.085	0.590
	ISS=1		0.311	0.426	0.991
	ISS=0		0.000	0.023	0.518
Blackrock	All	0.401	0.009	0.035	0.775
	ISS=1		0.055	0.187	0.997
	ISS=0		0.000	0.007	0.734
State Street	All	0.644	1.000	0.997	0.535
	ISS=1		1.000	1.000	0.965
	ISS=0		1.000	0.996	0.457

likely reflects that other investors' signals are weak or negative, pulling down the posterior expectation of proposal quality.

Taken together, these results emphasize that support rates are equilibrium outcomes shaped by information and strategic inference, not mechanical reflections of preference. Even with estimated preferences δ_i held fixed, differences in information precision and pivotal conditioning generate large cross-sectional variation in observed support. In particular, the strategic case (Case 4) reveals the core insight of the model: a higher pro-management preference does not necessarily translate into a higher support rate, because being pivotal is itself informative about the underlying state. Thus, interpreting raw support rates as preference indicators is fundamentally misleading—what appears as "pro-management voting" may, in equilibrium, simply reflect the information structure and the strategic logic of conditional pivotality.

6. Conclusion

This paper develops a structural framework to study how information and preferences jointly shape shareholder voting. We extend the original model of strategic voting by introducing both a public signal and shareholder-specific private signals, allowing for a transparent distinction between information that is commonly observed and information that is privately held. In equilibrium, shareholders vote strategically, internalizing the fact that their vote is pivotal only in states where the proposal quality is close to the passing threshold. This strategic conditioning implies that a higher observed support rate can arise from stronger information precision or pivotal selection, rather than from a more pro-management bias.

Using data on Say-on-Pay votes, the model fits the conditional distributions of dispersed shareholders' support rates remarkably well and reproduces each blockholder's conditional support rate under different ISS recommendations almost exactly. The estimated parameters indicate that dispersed shareholders rely heavily on the public signal, while the large blockholders possess more precise private information. Once strategic behavior is taken into account, we find that Vanguard and BlackRock's high support rates are consistent with moderate or even lower pro-management bias, whereas State Street exhibits a modest positive stance.

Overall, the results demonstrate that observed voting behavior reflects a mixture of information, preferences, and strategic inference. Understanding how these components interact is essential for interpreting institutional investors' influence on corporate governance and for evaluating potential regulatory or policy interventions.

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Appendix A: Proofs and Derivations

A.1 Preliminaries and notation

Latent quality, ownership, and voting. Each proposal j has latent quality $x_j \sim \mathcal{N}(0,1)$. Blockholders are indexed by $b \in \{1,\dots,N_B\}$ with ownership shares ψ_b, and dispersed shareholders collectively own $\Psi_D = 1 - \sum_{b=1}^{N_B} \psi_b$. Let $V_{Bj} = (V_{1j},\dots,V_{N_B j}) \in \{0,1\}^{N_B}$ denote the blockholder vote profile and $\Psi_{\text{yes}}(V_{Bj}) = \sum_b \psi_b V_{bj}$ the yes-shares from blockholders. A proposal passes if the total support rate exceeds the threshold λ^*.

Signals. A public signal $\eta_j = x_j + u_j$ is observed by all shareholders, with $u_j \sim \mathcal{N}(0,\sigma_u^2)$. Each shareholder i receives a private signal $z_{ij} = x_j + \varepsilon_{ij}$, independent across i and j and independent of u_j. For dispersed shareholders, $\varepsilon_{Dj} \sim \mathcal{N}(0,\sigma_D^2)$; for blockholder b, $\varepsilon_{bj} \sim \mathcal{N}(0,\sigma_b^2)$.

Cutoff strategies and Gaussian updating. Let k_D (resp. k_b) denote the dispersed (resp. blockholder b) cutoff in posterior-mean space. With Gaussian prior and signals, shareholder i's posterior mean is

$$m_{ij} \;=\; \mathrm{E}[x_j \mid \eta_j, z_{ij}] \;=\; \frac{\eta_j/\sigma_u^2 + z_{ij}/\sigma_i^2}{\Lambda_i}, \qquad \Lambda_i \equiv 1 + \frac{1}{\sigma_u^2} + \frac{1}{\sigma_i^2},$$

where $\sigma_i = \sigma_D$ for dispersed and $\sigma_i = \sigma_b$ for blockholder b. Shareholder i votes *For* iff $m_{ij} \ge k_i$.

Implied yes probabilities (conditional on (x_j, η_j)). Equivalently, these cutoffs imply threshold inequalities in z_{ij}:

$$m_{ij} \ge k_i \iff z_{ij} \;\ge\; \sigma_i^2\left(\Lambda_i k_i - \frac{\eta_j}{\sigma_u^2}\right).$$

Since $z_{ij} \mid x_j \sim \mathcal{N}(x_j, \sigma_i^2)$, the conditional yes probability is

$$\Pr(V_{Dj} = 1 \mid x_j, \eta_j) = \Phi\!\left(\frac{x_j - \sigma_D^2(\Lambda_D k_D - \eta_j/\sigma_u^2)}{\sigma_D}\right) = \Phi\!\left(\sigma_D\left[\tfrac{\eta_j}{\sigma_u^2} + \tfrac{x_j}{\sigma_D^2} - \Lambda_D k_D\right]\right), \tag{A.1}$$

$$\Pr(V_{bj} = 1 \mid x_j, \eta_j) = \Phi\!\Big(A_b\eta_j + B_b x_j - C_b\Big), \qquad A_b = \frac{\sigma_b}{\sigma_u^2}, \quad B_b = \frac{1}{\sigma_b}, \quad C_b = \sigma_b\Lambda_b k_b, \quad \Lambda_b = 1 + \frac{1}{\sigma_u^2} + \frac{1}{\sigma_b^2} \tag{A.2}$$

Blockholder profile likelihood. Conditional on (x_j, η_j) and independence of private noises across blockholders,

$$\Pr\big(V_{Bj} \mid x_j, \eta_j\big) = \prod_{b=1}^{N_B} \Big[\Phi\big(A_b \eta_j + B_b x_j - C_b\big)\Big]^{V_{Bj}(b)} \Big[1 - \Phi\big(A_b \eta_j + B_b x_j - C_b\big)\Big]^{1-V_{Bj}(b)}. \quad \text{(A.3)}$$

Notation. ϕ and Φ denote the standard normal pdf and cdf; for any $\tau \in (0,1)$, write $q = \Phi^{-1}(\tau)$.

A.2 Pivotality events with a large dispersed electorate

Dispersed shareholder pivotality. Fix a blockholder profile V_{Bj}. When the focal dispersed shareholder (FDS) is pivotal, the dispersed block must contribute precisely the residual share needed to hit λ^*:

$$\tau^{V_{Bj}} \equiv \frac{\lambda^* - \Psi_{\text{yes}}(V_{Bj})}{\Psi_D} \in (0,1), \qquad \Psi_{\text{yes}}(V_{Bj}) = \sum_b \psi_b V_{bj}. \quad \text{(A.4)}$$

Let $p_D(x_j, \eta_j)$ denote (A.1). For the FDS to be pivotal in the large-electorate limit, we must have

$$p_D(x_j, \eta_j) = \tau^{V_{Bj}}. \quad \text{(A.5)}$$

Because p_D is strictly increasing in x_j, there is a unique *knife-edge* quality $x_{\eta_j}^{V_{Bj}}$ solving (A.5):

$$x_{\eta_j}^{V_{Bj}} = \sigma_D^2 \left(\Lambda_D k_D - \frac{\eta_j}{\sigma_u^2} + \frac{q^{V_{Bj}}}{\sigma_D} \right), \qquad q^{V_{Bj}} \equiv \Phi^{-1}\big(\tau^{V_{Bj}}\big). \quad \text{(A.6)}$$

Blockholder b pivotality. Fix $V_{B,-b,j}$ (other blockholders' votes). The blockholder b is pivotal when the *dispersed* share places the total (excluding b) in the window $[\lambda^* - \psi_b,\ \lambda^*]$:

$$\tau_L^{V_{B,-b,j}} = \frac{\lambda^* - \Psi_{\text{yes}}(V_{B,-b,j}) - \psi_b}{\Psi_D}, \qquad \tau_H^{V_{B,-b,j}} = \frac{\lambda^* - \Psi_{\text{yes}}(V_{B,-b,j})}{\Psi_D}, \quad \text{(A.7)}$$

so pivotality requires

$$p_D(x_j, \eta_j) \in \big[\tau_L^{V_{B,-b,j}},\ \tau_H^{V_{B,-b,j}}\big]. \quad \text{(A.8)}$$

Monotonicity of p_D in x_j implies an equivalent x-window

$$x_{L,\eta_j}^{V_{B,-b,j}} = \sigma_D^2\left(\Lambda_D k_D - \frac{\eta_j}{\sigma_u^2} + \frac{\Phi^{-1}(\tau_L^{V_{B,-b,j}})}{\sigma_D}\right), \quad x_{H,\eta_j}^{V_{B,-b,j}} = \sigma_D^2\left(\Lambda_D k_D - \frac{\eta_j}{\sigma_u^2} + \frac{\Phi^{-1}(\tau_H^{V_{B,-b,j}})}{\sigma_D}\right).$$

A.3 Large-electorate limit and change-of-variable (Dirac delta & Jacobian)

We now formalize the $N_D \to \infty$ limit that collapses the pivotality event for the dispersed block.

Step 1: Write posteriors with finite N_D. For a *dispersed* focal voter,

$$f\left(x_j \mid z_{Dj}, \eta_j, \mathrm{PIV}_{Dj}\right) \;\propto\; f(x_j)\, f(z_{Dj} \mid x_j) \sum_{V_{Bj}} \Pr(V_{Bj} \mid x_j, \eta_j)\, \Pr(\mathrm{PIV}_{Dj} \mid x_j, \eta_j, V_{Bj}). \tag{A.9}$$

Let M_{-d} be the number of *other* dispersed yes votes; $M_{-d} \sim \mathrm{Bin}(N_D - 1,\ p_D(x_j, \eta_j))$. Given V_{Bj}, the FDS is pivotal iff the dispersed support rate (excluding the FDS) equals the threshold $\tau^{V_{Bj}}$, which is equivalent to $M_{-d} = (N_D - 1)\tau^{V_{Bj}}$ (up to the integer rounding implicit in the share standard).[4] Hence

$$\Pr(\mathrm{PIV}_{Dj} \mid x_j, \eta_j, V_{Bj}) \;=\; \Pr\left(M_{-d} = (N_D - 1)\tau^{V_{Bj}} \,\middle|\, x_j, \eta_j\right).$$

Step 2: Local-limit (Chamberlain–Rothschild / Myatt). Fix (x_j, η_j) and V_{Bj}, and denote $p = p_D(x_j, \eta_j)$ and $\tau = \tau^{V_{Bj}}$. By Stirling's formula, the binomial pmf is uniformly approximated by a Gaussian kernel of width $O(N_D^{-1/2})$ around $p = \tau$:

$$\Pr\left(M_{-d} = (N_D - 1)\tau \mid x_j, \eta_j\right) \;=\; \sqrt{\frac{1}{2\pi(N_D - 1)\tau(1 - \tau)}}\, \exp\left(-\tfrac{(N_D-1)(p-\tau)^2}{2\tau(1-\tau)}\right)[1 + o(1)].$$

Thus, as $N_D \to \infty$, this kernel converges in the *distributional* sense to a Dirac delta concentrated on the constraint $p = \tau$:

$$\Pr(\mathrm{PIV}_{Dj} \mid x_j, \eta_j, V_{Bj}) \;\Longrightarrow\; \kappa(\tau)\, \delta\left(p_D(x_j, \eta_j) - \tau\right), \qquad \text{for some scalar } \kappa(\tau) > 0.$$

[4]The $\leq$ / $\geq$ one-vote slack vanishes in the limit and does not affect the argument

The multiplicative scalar is absorbed by the posterior's normalizing constant.

Step 3: Change-of-variable from p_D to x. Use the identity $\delta(g(x)) = \sum_{x^*: g(x^*)=0} \delta(x - x^*)/|g'(x^*)|$ with $g(x) = p_D(x, \eta_j) - \tau$. Because p_D is strictly increasing in x, there is a unique root $x^* = x_{\eta_j}^{V_{Bj}}$ given by (A.6). Differentiating (A.1) in x,

$$\frac{\partial}{\partial x} p_D(x, \eta_j) = \phi\big(\sigma_D[\eta_j/\sigma_u^2 + x/\sigma_D^2 - \Lambda_D k_D]\big) \cdot \frac{1}{\sigma_D} \quad \Rightarrow \quad \left.\frac{\partial p_D}{\partial x}\right|_{x = x_{\eta_j}^{V_{Bj}}} = \frac{\phi(q^{V_{Bj}})}{\sigma_D}.$$

Therefore

$$\delta\big(p_D(x_j, \eta_j) - \tau^{V_{Bj}}\big) = \frac{\sigma_D}{\phi(q^{V_{Bj}})} \delta\big(x_j - x_{\eta_j}^{V_{Bj}}\big), \qquad \chi^{V_{Bj}} \equiv \frac{\sigma_D}{\phi(q^{V_{Bj}})}. \tag{A.10}$$

Step 4: Dispersed posterior in the limit (constraint form and evaluated form). Substituting the limit and (A.10) into (A.9) yields the *constraint form*:

$$\lim_{N_D \to \infty} f\big(x_j \mid z_{Dj}, \eta_j, \mathrm{PIV}_{Dj}\big) \propto f(x_j) \sum_{V_{Bj}} \underbrace{f(z_{Dj} \mid x_j)}_{\text{signal effect}} \underbrace{\Pr(V_{Bj} \mid x_j, \eta_j)}_{\text{strategic effect}} \underbrace{\delta\big(p_D(x_j, \eta_j) - \tau^{V_{Bj}}\big)}_{\text{pivotal constraint}}. \tag{A.11}$$

Applying (A.10) collapses the integral over x_j onto the knife-edge points (A.6) and introduces the Jacobian $\chi^{V_{Bj}}$:

$$f\big(x_j \mid z_{Dj}, \eta_j, \mathrm{PIV}_{Dj}\big) \propto \sum_{V_{Bj}} \underbrace{f\big(x_{\eta_j}^{V_{Bj}}\big)}_{\text{prior}} \underbrace{f\big(z_{Dj} \mid x_{\eta_j}^{V_{Bj}}\big)}_{\text{signal effect}} \underbrace{\chi^{V_{Bj}}}_{\text{Jacobian}} \underbrace{\Pr\big(V_{Bj} \mid x_{\eta_j}^{V_{Bj}}, \eta_j\big)}_{\text{strategic effect}} \delta\big(x_j - x_{\eta_j}^{V_{Bj}}\big). \tag{A.12}$$

Equations (A.11)–(A.12) constitute the new version of equation (8).

Blockholder posterior in the limit. Analogously, for blockholder b we have for finite N_D:

$$f\big(x_j \mid z_{bj}, \eta_j, \mathrm{PIV}_{bj}\big) \propto f(x_j) f(z_{bj} \mid x_j) \sum_{V_{B,-b,j}} \Pr(V_{B,-b,j} \mid x_j, \eta_j) \Pr(\mathrm{PIV}_{bj} \mid x_j, \eta_j, V_{B,-b,j}).$$

The pivotality probability converges to the *indicator* of the x-window induced by (A.8), because the dispersed share concentrates on $p_D(x_j, \eta_j)$ and the event is an interval (not a point). Hence

$$\lim_{N_D\to\infty} f\big(x_j \mid z_{bj}, \eta_j, \mathrm{PIV}_{bj}\big) \;\propto\; f(x_j)\, \underbrace{f(z_{bj} \mid x_j)}_{\text{signal effect}} \sum_{V_{B,-b,j}} \underbrace{\mathbf{1}\Big\{\, p_D(x_j, \eta_j) \in [\tau_L^{V_{B,-b,j}}, \tau_H^{V_{B,-b,j}}]\,\Big\}}_{\text{pivotal window in } x} \underbrace{\Pr\big(V_{B,-b,j} \mid x_j, \eta_j\big)}_{\text{strategic effect}}. \tag{A.13}$$

Equation (A.13) is the new version of equation (10).

Single-blockholder specialization. If $N_B = 1$ with ownership ψ_b,

$$\tau_L = \frac{\lambda^* - \psi_b}{\Psi_D}, \qquad \tau_H = \frac{\lambda^*}{\Psi_D}, \qquad x_{L,\eta_j} = \sigma_D^2\left(\Lambda_D k_D - \frac{\eta_j}{\sigma_u^2} + \frac{\Phi^{-1}(\tau_L)}{\sigma_D}\right), \quad x_{H,\eta_j} = \sigma_D^2\left(\Lambda_D k_D - \frac{\eta_j}{\sigma_u^2} + \frac{\Phi^{-1}(\tau_H)}{\sigma_D}\right.$$

Then the pivotal window in (A.13) becomes $x_j \in [x_{L,\eta_j}, x_{H,\eta_j}]$.

A.4 Summary of objects used in (A.12)–(A.13)

- Dispersed yes probability: $p_D(x_j, \eta_j) = \Phi\big(\sigma_D[\eta_j/\sigma_u^2 + x_j/\sigma_D^2 - \Lambda_D k_D]\big)$, with $\Lambda_D = 1 + 1/\sigma_u^2 + 1/\sigma_D^2$.
- Blockholder yes probability: $\Pr(V_{bj} = 1 \mid x_j, \eta_j) = \Phi(A_b\eta_j + B_b x_j - C_b)$ with $A_b = \sigma_b/\sigma_u^2$, $B_b = 1/\sigma_b$, $C_b = \sigma_b\Lambda_b k_b$, $\Lambda_b = 1 + 1/\sigma_u^2 + 1/\sigma_b^2$.
- Pivotal dispersed share: $\tau^{V_{Bj}} = (\lambda^* - \Psi_{\text{yes}}(V_{Bj}))/\Psi_D$.
- Knife-edge quality: $x_{\eta_j}^{V_{Bj}} = \sigma_D^2\big(\Lambda_D k_D - \eta_j/\sigma_u^2 + q^{V_{Bj}}/\sigma_D\big)$ with $q^{V_{Bj}} = \Phi^{-1}(\tau^{V_{Bj}})$.
- Jacobian (delta change-of-variable): $\chi^{V_{Bj}} = \sigma_D/\phi(q^{V_{Bj}})$.
- Blockholder pivotal window: $\tau_{L,H}^{V_{B,-b,j}}$ as in (A.7); indicator $\mathbf{1}\{p_D(x_j, \eta_j) \in [\tau_L, \tau_H]\}$.